College Algebra, Section 2.3, #58
Systems of Linear Equations in Two Variables

Nutrition *Each ounce of substance A supplies 6% of a nutrient a patient needs, and each ounce of substance B supplies 10% of the required nutrient. If the total number of ounces given to the patient was 14 and 100% of the nutrient was supplied, how many ounces of each substance was given?*[1]

You probably recognize this as a "mixture" problem. Since we have two variables, ounces of substance A and ounces of substances B, we can set up two equations: one that represents quantity and the other that represents value.

Some people like to draw a chart but it's just as easy to write the two equations where A is the number of ounces of substance A and B is the number of ounces of substance B.

Quantity: $A + B = 14$

Value: $0.06A + 0.10B = 1.00$

Now that we have the two equations and because the problem didn't specify a method, it's up to you to choose a method for solving. Currently, there are three choices: Substitution, Elimination, or Graphing. All three are shown below.

Substitution

Solve one equation for one variable and substitute into the other equation.

I've chosen to solve for A in the first equation.

$$\begin{aligned} A + B &= 14 \\ A &= 14 - B \end{aligned}$$

Substitute this value of A into the second equation.

$$\begin{aligned} 0.06A + 0.10B &= 1.00 \\ 0.06(14 - B) + 0.10B &= 1.00 \\ 0.84 - 0.06B + 0.10B &= 1.00 \\ 0.84 + .04B &= 1.00 \\ 0.04B &= 0.16 \\ B &= 4 \end{aligned}$$

Back-substitute $B = 4$ to find the value of A.

$$\begin{aligned} A &= 14 - B \\ A &= 14 - 4 \\ A &= 10 \end{aligned}$$

The patient was given 10 ounces of substance A and 4 ounces of substance B.

Elimination

Multiply one equation by a number that will result in the coefficients of one variable being equal but opposite.

I've chosen to multiply the first equation by -0.06.

$$\begin{aligned} -0.06(A + B) &= -0.06(14) \\ -0.06A - 0.06B &= -0.84 \end{aligned}$$

[1] Harshbarger/Yocco, *College Algebra In Context, 5e*, p. 138, #58.

Now the two equations look like this:

$$\begin{aligned} -0.06A &- 0.06B &&= -0.84 \\ 0.06A &- 0.10B &&= 1.00 \end{aligned}$$

The coefficients of A are equal but opposite and adding the two equations vertically will eliminate A.

$$\begin{array}{rrrrr} & -0.06A & - & 0.06B & = & -0.84 \\ + & 0.06A & + & 0.10B & = & 1.00 \\ \hline & & & 0.04B & = & 0.16 \\ & & & B & = & 4 \end{array}$$

Back-substitute $B = 4$ into our equation for "Quantity" to find the value of A.

$$\begin{aligned} A + B &= 14 \\ A + 4 &= 14 \\ A &= 10 \end{aligned}$$

The patient was given 10 ounces of substance A and 4 ounces of substance B.

Graphing

Solve each equation for one variable. The variable you solve for is your choice but I'm going to solve for B so that A is the independent variable and B is the dependent variable. This will give an (A,B) pair as the solution.

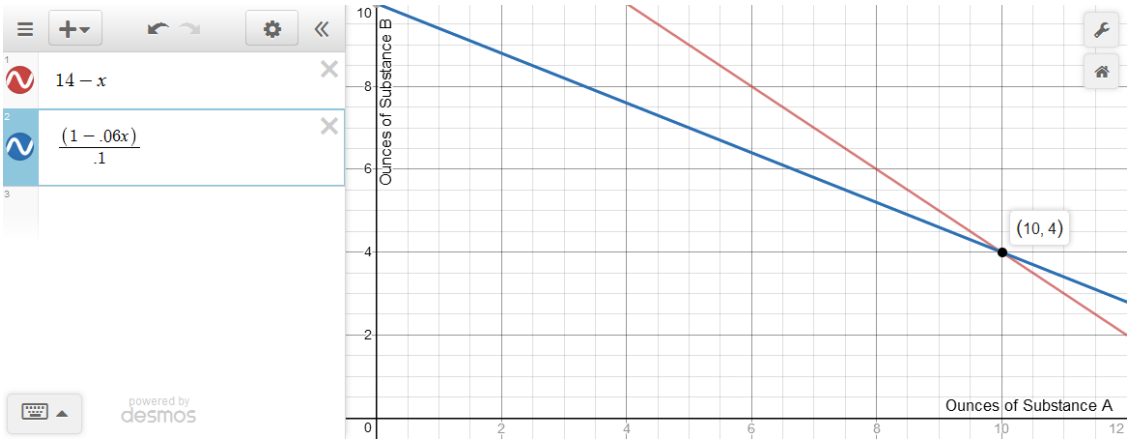
The first equation becomes...

$$\begin{aligned} A + B &= 14 \\ B &= 14 - A \end{aligned}$$

And the second equation becomes...

$$\begin{aligned} 0.06A + 0.10B &= 1.00 \\ 0.10B &= 1.00 - 0.06A \\ B &= \frac{1.00 - 0.06A}{0.10} \end{aligned}$$

Graph the two equations with A on the horizontal axis and B on the vertical axis and look for the intersection of the two lines.



These two lines intersect at the point $(10, 4)$. So patient was given 10 ounces of substance A and 4 ounces of substance B.