U.S. Population *The U.S. population can be modeled by the function* $y = 165.6x^{1.345}$*, where* y *is in thousands and* x *is the number of years after 1800.*[1]

a. *What was the population in 1960, according to this model?*

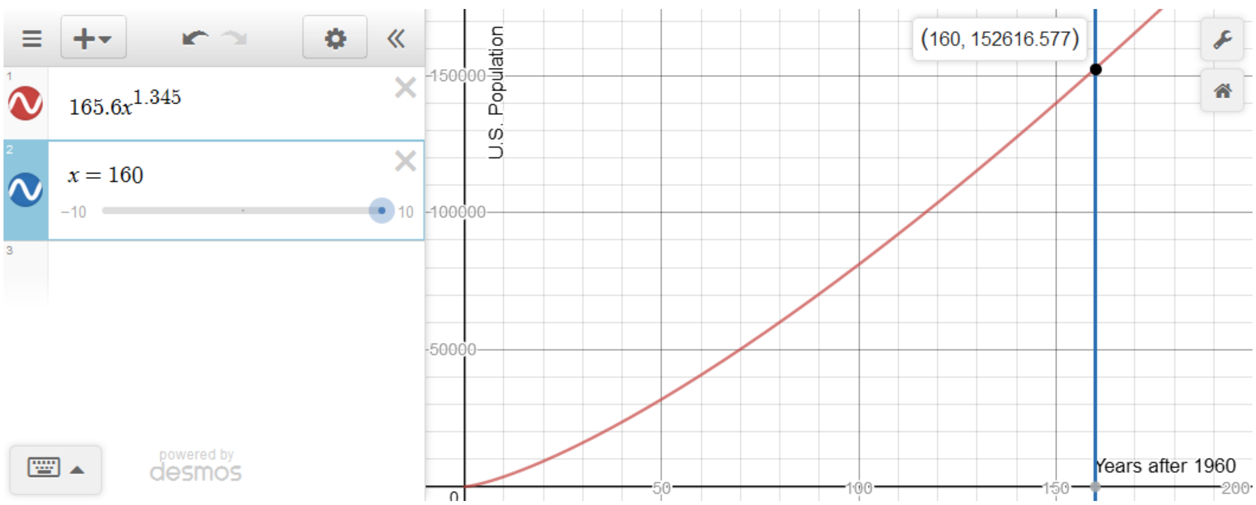
We can approach this problem either algebraically or graphically. I'll show both:

Algebraically, notice that the year 1960 is 160 years after the year 1800. Find y, the U.S. population in 1960, by letting $x = 160$ and solving for y, the population in thousands.

$$\begin{aligned} y &= 165.6\,x^{1.345} \\ &= 165.6\,(160)^{1.345} \\ &\approx 165.6\,(921.55976882) && \text{I kept all of these decimal places to minimize round-off error.} \\ &\approx 152616.5772 \\ &\approx 152617 \end{aligned}$$

But y is in thousands and according to this model the U.S. population in 1960 was 152,617,000 people.

To do find the answer graphically, we graph $y = 165.6x^{1.345}$ and find the vale of y when $x = 160$.



Again, y is in thousands and according to this model the U.S. population in 1960 was 152,617,000 people.

b. *Is the graph of this function concave up or concave down? What does this mean?*

The graph is concave up and we can say that the function is increasing at an increasing rate.

[1] Harshbarger/Yocco, *College Algebra In Context, 5e*, p. 211, #52.

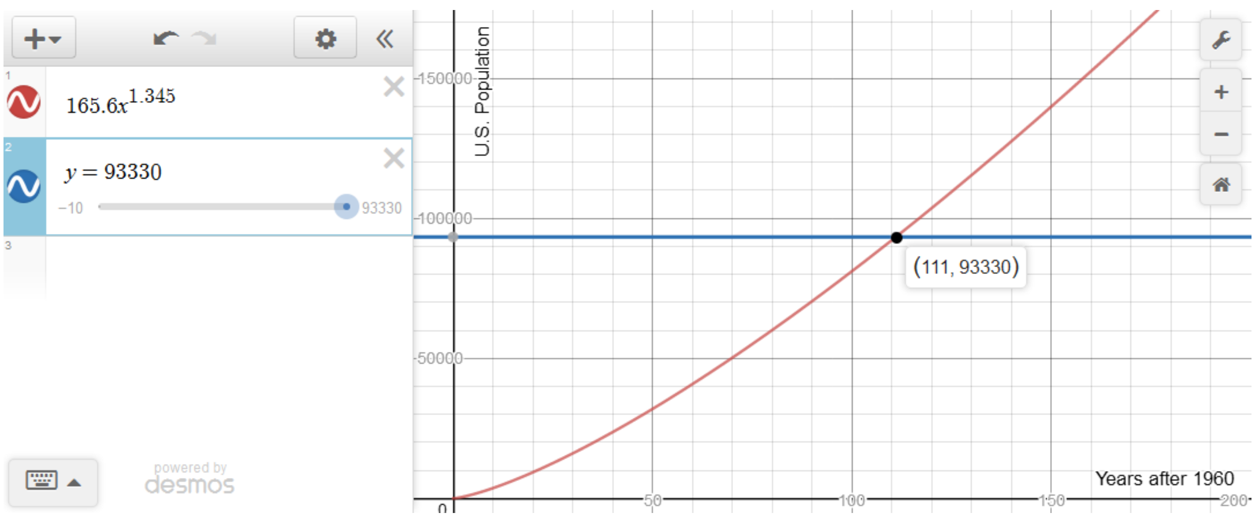
c. *Use numerical or graphical methods to find when the model estimates the population to be 93,330,000.*

Here we are given the population (a y-value) and asked to find the year (an x-value). Solving this algebraically requires the use of logarithms and looks like this:

$$\begin{aligned} y &= 165.6\,x^{1.345} \\ 93{,}330 &= 165.6\,x^{1.345} && \text{Remember, } y \text{ was given in thousands.} \\ 563.587 &= x^{1.345} \\ log\,563.587 &= log\,x^{1.345} \\ 2.76096 &= 1.345\,log\,x \\ 2.04532 &= log\,x \\ 10^{2.04532} &= x \\ 111 &= x \end{aligned}$$

x is the number of years after 1800 so we can say that according to this model the U.S. population was 93,330,000 in 1911.

I think solving this graphically is much easier than solving algebraically. We are given a y-value and we just have to look for the intersection between the given curve and the horizontal line to determine the value of x.



The x-coordinate of the point of intersection is the number of years after 1800 so we can say that according to this model the U.S. population was 93,330,000 in 1911.