Cost-Benefit *Suppose for a certain city the cost C of obtaining drinking water that contains p% impurities (by volume) is given by* [1]

$$C = \frac{120{,}000}{p} - 1200$$

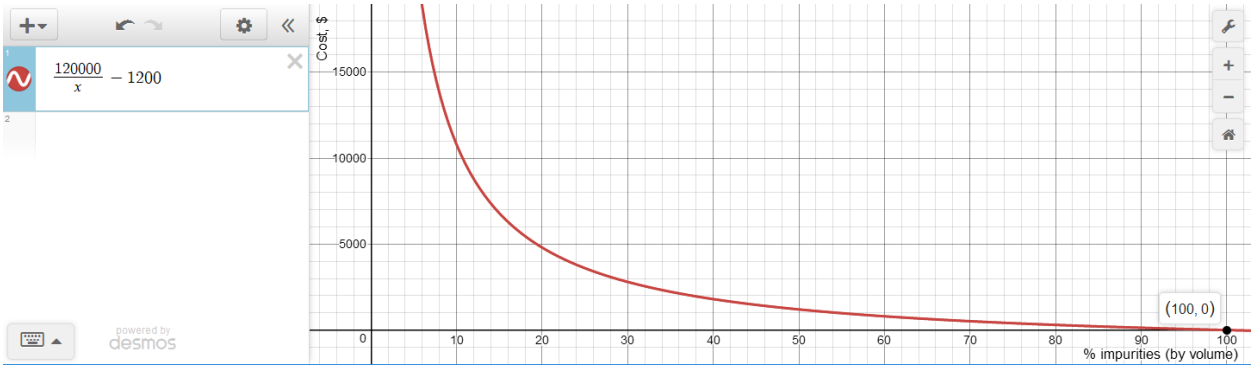
a. *What is the cost of drinking water that is 100% impure?*

If the water is 100% impure, then $p = 100$ and we can substitute this value into the given equation to find the cost, C.

$$\begin{aligned} C &= \frac{120{,}000}{p} - 1200 \\ &= \frac{120{,}000}{100} - 1200 \\ &= 1200 - 1200 \\ &= 0 \end{aligned}$$

The cost of drinking water that is 100% impure is \$0.

We can also see this from the graph of C.



The point $(100, 0)$ tells us that when the impurities are at 100%, the cost of the drinking water is \$0.

b. *What is the cost of drinking water that is 50% impure?*

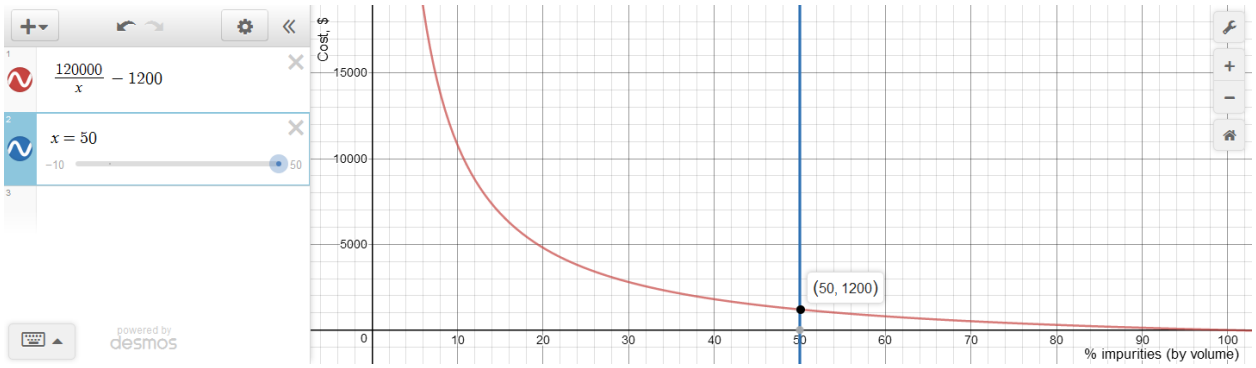
Similar to part (a), if the water is 50% impure, then $p = 50$ and we substitute this value into the given equation to find the cost, C.

$$\begin{aligned} C &= \frac{120{,}000}{p} - 1200 \\ &= \frac{120{,}000}{50} - 1200 \\ &= 2400 - 1200 \\ &= 1200 \end{aligned}$$

The cost of drinking water that is 50% impure is \$1,200.

[1] Harshbarger/Yocco, *College Algebra In Context, 5e*, p. 262, #70.

Again, this is verified by the graph of C.



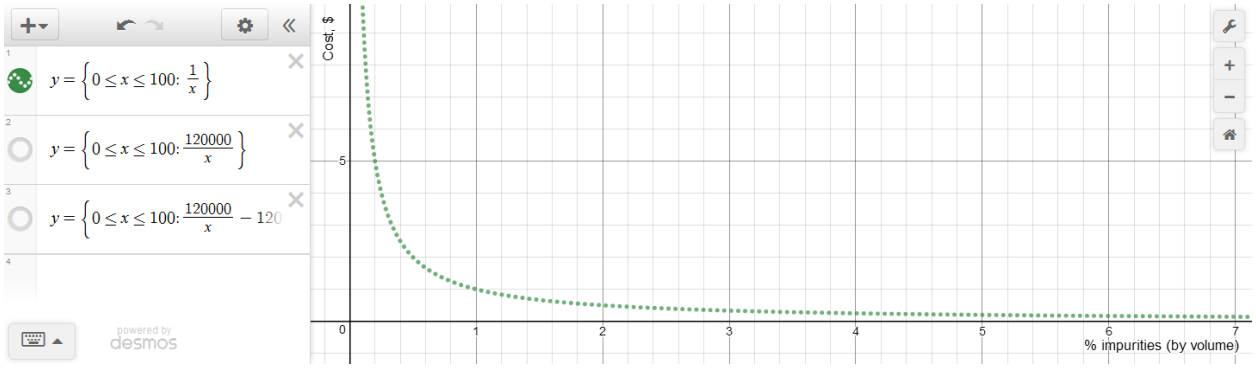
The point $(50, 1200)$ tells us that when the impurities are at 50%, the cost of the drinking water is \$1200.

c. *What transformations of the graph of the reciprocal function give the graph of this function?*

You might notice that I've restricted the domain of the following graphs to values between 0 and 100, inclusive. I did this for a couple of reasons: (1) in the context of this problem, the domain is restricted between 0% impurities and 100% impurities and (2) this restriction helps us focus on the half of the graph that appears in Quadrant I as there is really no reason to look at Quadrant III.

Let's start with the graph of $y = \frac{1}{x}$ (the reciprocal function) and transform it to get the graph of $y = \dfrac{120{,}000}{x} - 1200$.

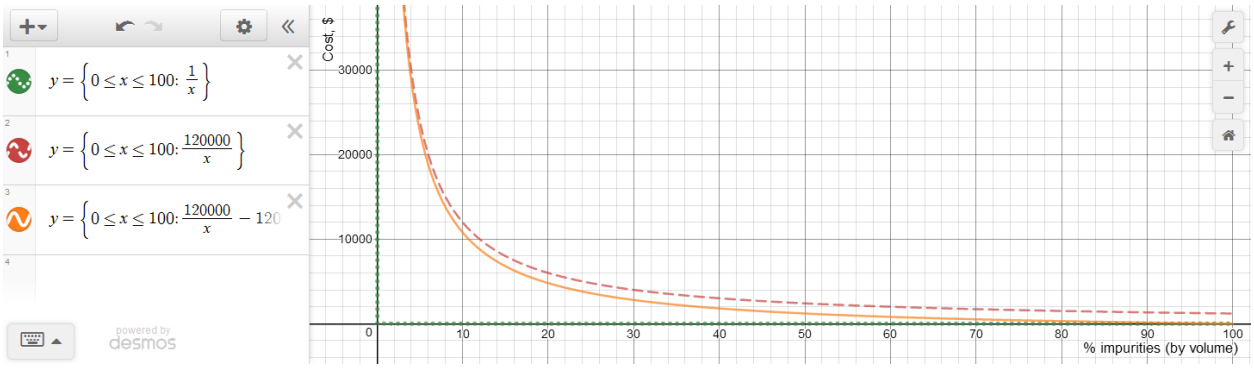
$y = \frac{1}{x}$ The basic function. Shown in green dots.



For the next two transformations, I had to change the scale on both axes so we can clearly see the complete graph.

$y = \dfrac{120000}{x}$ Notice how $\frac{1}{x}$ was multiplied by 120,000. This gives a vertical stretch by a factor of 120,000. Shown in red dashes.

$y = \dfrac{120{,}000}{x} - 1200$ Vertical shift down 1200 units. Shown in solid orange.



In short, the transformations are: a vertical stretch using a factor of 120,000 and a vertical shift down of 1200 units.