

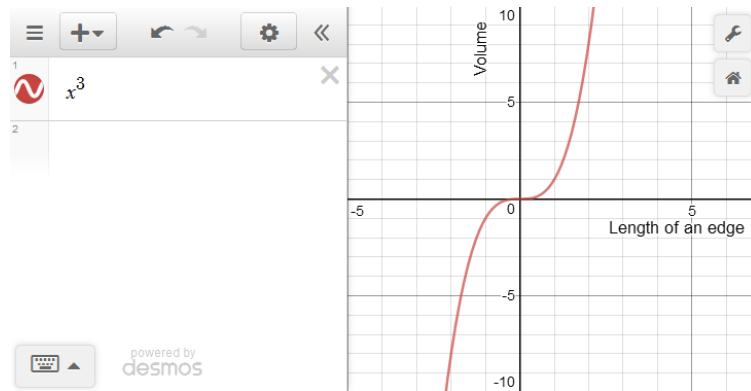
College Algebra, Section 4.3, #44
 One-to-One and Inverse Functions

Volume of a Cube The volume of a cube is $f(x) = x^3$ cubic inches, where x is the length of the edge of the cube in inches.¹

a. Is this function one-to-one?

A function f is one-to-one if each output of the function corresponds to exactly one input in the domain of the function.

A function is one-to-one if no horizontal line can intersect the graph of the function in more than one point. Let's look at the graph of $f(x) = x^3$.

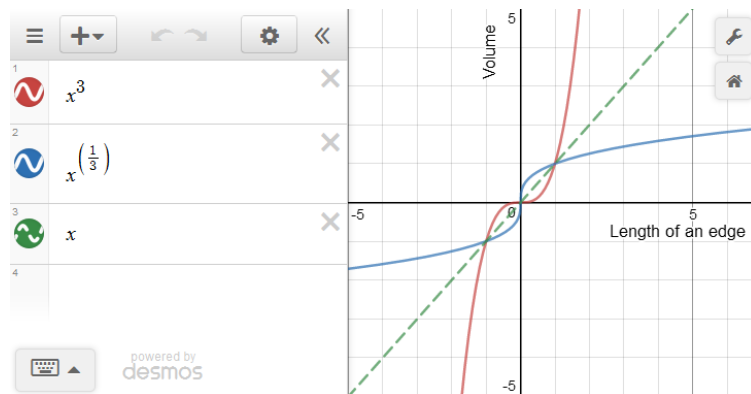


Since any horizontal line will intersect the graph of f only once, we can say that the function passes the horizontal line test and, yes, it is one-to-one.

b. Find the inverse of this function.

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|-------------------------------|---|
| $f(x) = x^3$ | Start with the original function, $f(x)$. |
| $y = x^3$ | Replace $d(x)$ with y . |
| $x = y^3$ | Replace all x 's with y 's and all y 's with x 's. |
| $\sqrt[3]{x} = \sqrt[3]{y^3}$ | Solve for y . |
| $\sqrt[3]{x} = y$ | |
| $f^{-1}(x) = x^{\frac{1}{3}}$ | Replace y with the inverse function notation, $f^{-1}(x)$. |

The inverse of $f(x) = x^3$ is $f^{-1}(x) = x^{\frac{1}{3}}$.



Notice how the graphs of f and f^{-1} are reflections of each other across the line $y = x$.

¹Harshbarger/Yocco, *College Algebra In Context*, 5e, p. 285, #44.