College Algebra, Section 4.4, #50
Additional Equations and Inequalities

Profit *The monthly profit from producing and selling x units of a product is given by*

$$P(x) = -0.01x^2 + 62x - 12{,}000$$

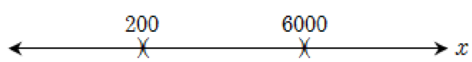
Producing and selling how many units will result in a profit for this product? [1]

We want to know what values of x will result in the profit being greater than zero. That is: $P(x) > 0$.

To solve this algebraically, we begin by solving the equality:

$$\begin{aligned} P(x) &= 0 \\ -0.01x^2 + 62x - 12{,}000 &= 0 \\ -0.01\left(x^2 - 6200x + 1{,}200{,}000\right) &= 0 && \text{Factor out } -0.01. \\ -0.01\,(x-200)(x-6000) &= 0 && \text{Complete factoring the quadratic and solve for } x. \\ x = 200 \quad \text{or} \quad x &= 6000 \end{aligned}$$

Remember, our goal is to find the values of x that make $P(x) > 0$ true. The values $x = 200$ and $x = 6000$ divide the number line into three intervals and it's our job to find which of these three intervals contain values of x that "work" in this inequality. That is, which values of x that result in a positive value for $-0.01\,(x-200)(x-6000)$.

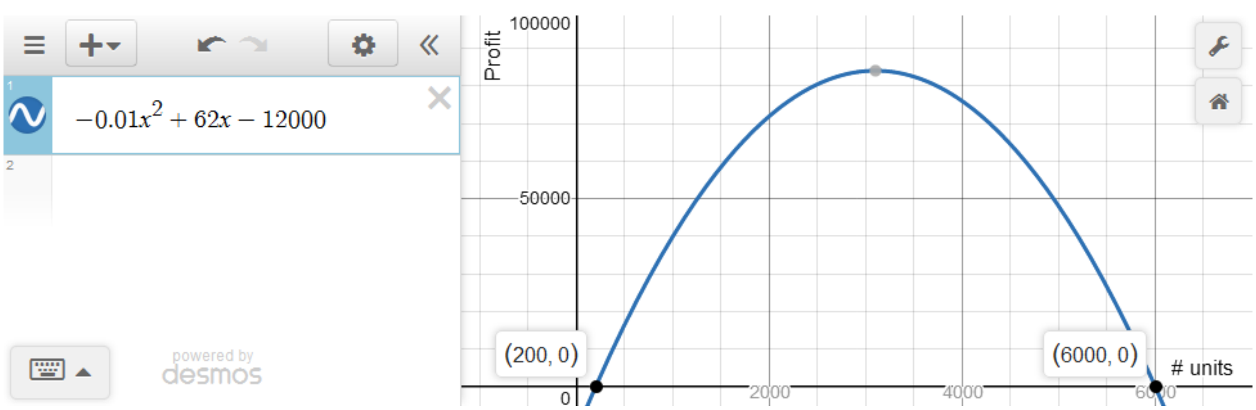


sign of $(x-200)$	$-$	$+$	$+$
sign of $(x-6000)$	$-$	$-$	$+$
sign of $(x-200)(x-6000)$	$+$	$-$	$+$
sign of $-0.01\,(x-200)(x-6000)$	$-$	$+$	$-$

The function $P(x) = -0.01\,(x-200)(x-6000)$ is positive in the interval $200 < x < 6000$, so the solution to $-0.01x^2 + 62x - 12{,}000 > 0$ is $200 < x < 6000$.

Producing and selling more than 200 but less than 6000 units of this product will result in a profit.

To answer this question graphically, we graph the original function P and find the values of x when P is ABOVE the x-axis. That is, $P(x) > 0$.



Again, the values of x where P is above the x-axis are $200 < x < 6000$.

[1] Harshbarger/Yocco, *College Algebra In Context*, 5e, p. 299, #50.