Doubling Time *The number of quarters needed to double an investment when a lump sum is invested at 8%, compounded quarterly, is given by* $n = log_{1.02} 2$. [1]

a. *Use the change of base formula to find* n,

On most calculators there is a *log* and a *ln* key. Remember, these are "log base 10" (the common log) and "log base e" (the natural log), respectively.

In this problem we are given "log base 1.02" and asked to use the change of base formula to solve for n.

The change of base formula looks like this:

Change-of-Base Formula

If $b > 0, b \neq 1, a > 0, a \neq 1$, and $x > 0$, then

$$\log_a x = \frac{\log_b x}{\log_b a}$$

In particular, for base 10 and base e,

$$\log_a x = \frac{\log x}{\log a} \quad \text{and} \quad \log_a x = \frac{\ln x}{\ln a}$$

Without getting lost in the details, you'll want to remember this: $log_a x = \dfrac{log_b x}{log_b a}$ where b is your choice.

You will almost always want to choose $b = 10$ or $b = e$ because these are the functions you already have on your calculator.

I'm going to use natural log but common log would work just as well.

$$\begin{aligned} n &= log_{1.02} 2 \\ &= \frac{ln\, 2}{ln\, 1.02} \\ &\approx 35.0027 \end{aligned}$$

The investment will take 35 quarters to double.

b. *In how many years will the investment double?*

In part (a) we found that it would take 35 quarters for the investment to double.

Each year has 4 quarters so it would take $\dfrac{35}{4}$, or 8.75, years for the investment to double.

[1] Harshbarger/Yocco, *College Algebra In Context, 5e*, p. 357, #74.