College Algebra, Section 6.4, #36
Polynomial Equations Continued; Fundamental Theorem of Algebra

Revenue *The revenue from the sale of a product is given by* $R = 250x - 5x^2 - x^3$. *If the sale of 5 units gives a total revenue of $1000, Use synthetic division to find another number of units that will give $1000 in revenue.* [1]

We want to know values of x that will result in $R = 1000$. Let's start by substituting $R = 1000$ and solving so one side of the equation equals 0.

$$\begin{aligned} R(x) &= 250x - 5x^2 - x^3 \\ 1000 &= 250x - 5x^2 - x^3 \\ 0 &= -1000 + 250x - 5x^2 - x^3 \\ 0 &= -x^3 - 5x^2 + 250x - 1000 \\ 0 &= x^3 + 5x^2 - 250x + 1000 \end{aligned}$$

Now that the equation is in descending order, we're ready to do some synthetic division.

We are given that $x = 5$ is a solution to the above equation (we say $x = 5$ is a root) so that's where we'll start.

$$\begin{array}{r|rrrr} 5 & 1 & 5 & -250 & 1000 \\ & & 5 & 50 & -1000 \\ \hline & 1 & 10 & -200 & 0 \end{array}$$

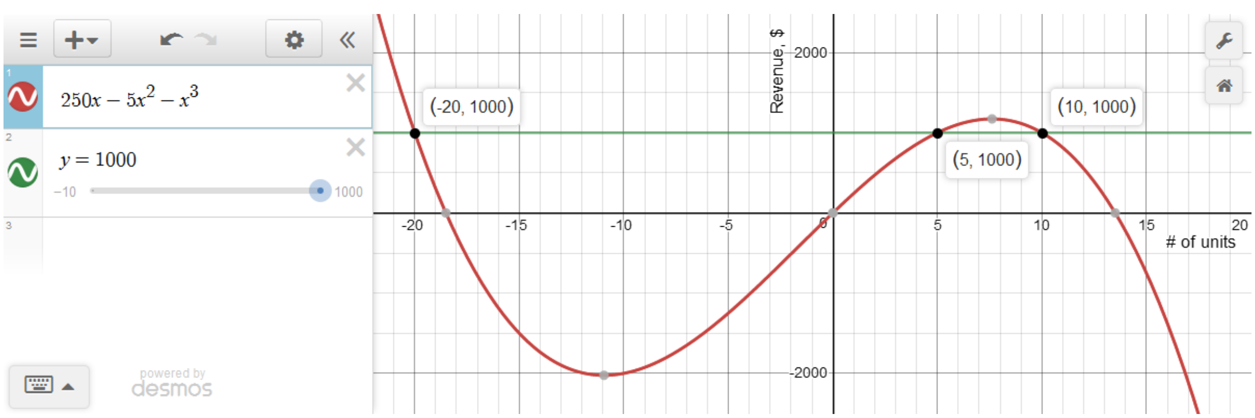
The quadratic factor, shown in the bottom row of the synthetic division, is $x^2 + 10x - 200$. To find the other solutions, set this factor equal to zero and solve for x.

$$\begin{aligned} x^2 + 10x - 200 &= 0 \\ (x + 20)(x - 10) &= 0 \end{aligned}$$

$$\begin{aligned} x + 20 = 0 \quad &\text{or} \quad x - 10 = 0 \\ x = -20 \quad &\text{or} \quad x = 10 \end{aligned}$$

Omitting the negative value of x (because we can't sell less than zero units), the level of sale that results in a total revenue of $1000 is 10 units.

This result can be verified by graphing the functions $R = 250x - 5x^2 - x^3$ and $y = 1000$ and finding the points of intersection of the two graphs.



[1] Harshbarger/Yocco, *College Algebra In Context, 5e*, p. 477, #36.