Loans *A bank loans \$285,000 to a development company to purchase three business properties. One of the properties costs \$45,000 more than another, and the third costs twice the sum of these two properties.* [1]

a. *Write an equation that represents the total loaned as the sum of the costs of the three propertied, if the costs are x, y, and z respectively.*

Regardless of the individual costs of the properties, the total cost is \$285,000.

This total is represented by the sum $x + y + z = 285{,}000$

b. *Write an equation that states that one cost is \$45,000 more than another.*

Let's say the first property, x, costs \$45,000 more than the second property, y, then $x = y + 45{,}000$.

c. *Write an equation that states that the third cost is equal to twice the sum of the other two.*

The cost of the third property, z, is two times the sum of the first, x, and the second, y. So, $z = 2(x + y)$.

d. *Solve the system of equations to find the cost of each property.*

From part (a), we have $x + y + z = 285{,}000$. Notice that this equation is in a form with all the variables on one side of the equal sign and a constant on the other side. This is the format we want for each of the equations in the system.

From part (b), we have $x = y + 45{,}000$ but we need to rearrange it into the correct form by subtracting y from each side: $x - y = 45{,}000$.

Finally, from part (c), $z = 2(x + y)$, we can subtract z from each side that results in $2x + 2y - z = 0$.

Now we have all three equations in the proper form and we can put them together into the following system:

$$\begin{cases} x + y + z = 285{,}000 \\ x - y = 45{,}000 \\ 2x + 2y - z = 0 \end{cases}$$

Begin by labeling the equations so we can keep track of what we're doing.

$$\begin{cases} x + y + z = 285{,}000 & \text{(A)} \\ x - y = 45{,}000 & \text{(B)} \\ 2x + 2y - z = 0 & \text{(C)} \end{cases}$$

The x in equation (A) already has a coefficient of 1 so we can use it to eliminate the x's in equations (B) and (C) as follows:

(B)−(A) → (B) and (C)−2(A) → (C)

$$\begin{cases} x + y + z = 285{,}000 & \text{(A)} \\ -2y - z = -240{,}000 & \text{(B)} \\ -3z = -570{,}000 & \text{(C)} \end{cases}$$

[1] Harshbarger/Yocco, *College Algebra In Context*, *5e*, p. 520 #32.

Multiply (C) by $-1/3$ to get 1 as the coefficient on the z.

$-\frac{1}{3}$ (C) $\rightarrow$ (C)

$$\begin{cases} x + y + z = 285{,}000 & \text{(A)} \\ -2y - z = -240{,}000 & \text{(B)} \\ z = 190{,}000 & \text{(C)} \end{cases}$$

Now we can use equation (C), $z = 190{,}000$, to back-substitute and find the value for y.

Using equation (B),

$$\begin{aligned} -2y - z &= -240{,}000 \\ -2y - 190{,}000 &= -240{,}000 \\ -2y &= -50{,}000 \\ y &= 25{,}000 \end{aligned}$$

And finally, we substitute the values of y and z into equation (A) to solve for x.

$$\begin{aligned} x + y + z &= 285{,}000 \\ x + 25{,}000 + 190{,}000 &= 285{,}000 \\ x + 215{,}000 &= 285{,}000 \\ x &= 70{,}000 \end{aligned}$$

The costs of the three properties are \$70,000, \$25,000, and \$190,000.

Another way we can achieve the same result is by using matrices. Begin with the augmented matrix...

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 285{,}000 \\ 1 & -1 & 0 & 45{,}000 \\ 2 & 2 & -1 & 0 \end{array}\right]$$

and use reduced row-echelon form (rref) on your calculator to get the solution matrix...

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 70{,}000 \\ 0 & 1 & 0 & 25{,}000 \\ 0 & 0 & 1 & 190{,}000 \end{array}\right]$$

Here we see that $x = 70{,}000$, $y = 25{,}000$, and $z = 190{,}000$ and the costs of the three properties are \$70,000, \$25,000, and \$190,000.