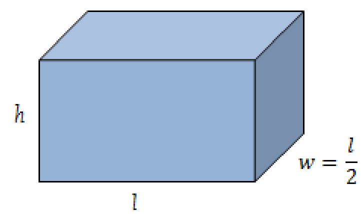
Constructing a Box *A box with a top is to be constructed with the width of its base equal to half of the length of its base. If it is to have volume 600 cubic centimeters and uses 2200 square centimeters of material, what are the dimensions of the box?* [1]

This calls for a picture of the situation.



We aren't given any information regarding the length or height of the box so I've assigned the variables l and h, respectively, to these measurements.

But we are given that "the width of its base equal to half of the length of its base" so we can say, $w = \frac{l}{2}$, where w is the width of the box.

Now we have two additional pieces of information: the volume of the box and the amount of material it takes to make the box. I'll call these V (for volume) and S (for surface area), respectively.

For volume...

$$l \cdot w \cdot h = V$$

$$l \cdot \frac{l}{2} \cdot h = 6000$$

$$\frac{l^2 h}{2} = 6000$$

$$l^2 h = 12{,}000$$

$$h = \frac{12{,}000}{l^2}$$

And for surface area, the box has the top and bottom that each have area lw, the front and back that each have area lh, and two ends that each have area wh.

This gives us...

$$2lw + 2lh + 2wh = S$$

$$2l \cdot \frac{l}{2} + 2lh + 2 \cdot \frac{l}{2} \cdot h = 2200$$

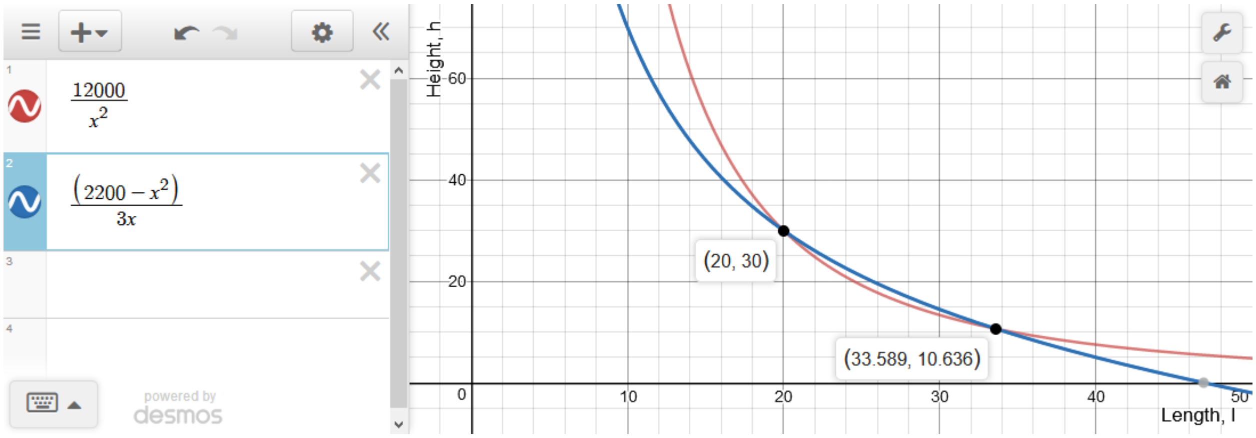
$$l^2 + 2lh + lh = 2200$$

$$3lh = 2200 - l^2$$

$$h = \frac{2200 - l^2}{3l}$$

[1]Harshbarger/Yocco, *College Algebra In Context*, 5e, p. 578, #36.

Now we have two equations, each with two unknowns and we can solve by graphing. The intersection of the V and S curves will be the (l,h) pair that satisfies all of the given constraints.



Two boxes are possible. One has dimensions 20cm long x 10cm wide x 30cm high and the other has dimensions that are approximately 30.59cm long x 16.79cm wide x 10.64cm high.