Find (a) the center (h, k) and radius r of the circle, (b) graph the circle, and (c) find the intercepts, if any.[1]

$$x^2 + y^2 + 4x + 2y - 20 = 0$$

If we can write the given equation in the standard form of an equation of a circle

$$(x - h)^2 + (y - k)^2 = r^2$$

then we can easily identify the center (h, k) and radius r. To write the equation in the standard form, we complete the square in x and then complete the square in y.

$$x^2 + y^2 + 4x + 2y - 20 = 0$$

Group the x's and y's

$$x^2 + 4x + \quad + y^2 + 2y \quad = 20$$

To complete the square in x, we compute one-half of the linear coefficient ($\frac{1}{2} \cdot 4 = 2$), then add the square of that result ($2^2 = 4$) to both sides

$$x^2 + 4x + 4 + y^2 + 2y \quad = 20 + 4$$

To complete the square in y, we compute one-half of the linear coefficient ($\frac{1}{2} \cdot 2 = 1$), then add the square of that result ($1^2 = 1$) to both sides

$$x^2 + 4x + 4 + y^2 + 2y + 1 = 20 + 4 + 1$$

Now factor the trinomials on the left side.

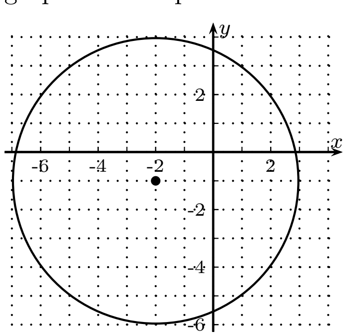
$$(x + 2)^2 + (y + 1)^2 = 25$$

This has the form

$$(x - -2)^2 + (y - -1)^2 = 5^2$$

So the center of the circle is $(-2, -1)$ and the radius is 5.

The graph of the equation is



[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 50, #26.

As always, to find the x-intercept(s), we substitute $y = 0$. Our work will be eased if we use the equation of the circle in standard form.

$$\begin{aligned} (x+2)^2 + (y+1)^2 &= 25 \\ (x+2)^2 + (0+1)^2 &= 25 \\ (x+2)^2 + (1)^2 &= 25 \\ (x+2)^2 + 1 &= 25 \\ (x+2)^2 &= 24 \end{aligned}$$

From the square root method (If $x^2 = p$, then $x = \sqrt{p}$ or $x = -\sqrt{p}$)

$$x + 2 = \sqrt{24} \text{ or } x + 2 = -\sqrt{24}$$

So

$$x = -2 + 2\sqrt{6} \text{ or } x = -2 - 2\sqrt{6}$$

Thus the x-intercepts are $\left(-2 + 2\sqrt{6}, 0\right)$ and $\left(-2 - 2\sqrt{6}, 0\right)$.

To find the y-intercept(s), we substitute $x = 0$. Our work will be eased if we use the equation of the circle in standard form.

$$\begin{aligned} (x+2)^2 + (y+1)^2 &= 25 \\ (0+2)^2 + (y+1)^2 &= 25 \\ (2)^2 + (y+1)^2 &= 25 \\ 4 + (y+1)^2 &= 25 \\ (y+1)^2 &= 21 \end{aligned}$$

From the square root method (If $x^2 = p$, then $x = \sqrt{p}$ or $x = -\sqrt{p}$)

$$y + 1 = \sqrt{21} \text{ or } y + 1 = -\sqrt{21}$$

So

$$y = -1 + \sqrt{21} \text{ or } y = -1 - \sqrt{21}$$

Thus the y-intercepts are $\left(0, -1 + \sqrt{21}\right)$ and $\left(0, -1 - \sqrt{21}\right)$.