Minimum Payments for Credit Cards *Holders of credit cards issed by banks, department stores, oil companies, and so on, receive bills each month that state minimum amounts that must be paid by a certain due date. The minimum due depends on the total amount owed. One such credit card company uses the following rules: For a bill of less than \$10, the entire amount is due. For a bill of at least \$10 but less than \$500, the minimum due is \$10. A minimum of \$30 is due on a bill of at least \$500 but less than \$1000, a minimum of \$50 is due on a bill of at least \$1000 but less than \$1500, and a minimum of \$70 is due on bills of \$1500 or more. Find a function f that describes the minimum payment due on a bill of x dollars. Graph f.*[1]

The next-to-last sentence of that long paragraph tells us our task: "Find a function f that describes the minimum payment due on a bill of x dollars." This means we want a function f whose input (independent variable) is the amount of the bill, x, in dollars, and whose output (dependent variable) is the amount, $f(x)$, in dollars, of the minimum payment.

$$\begin{aligned} \text{Let } x &= \text{ amount of bill, in dollars} \\ \text{and } f(x) &= \text{ minimum payment, in dollars} \end{aligned}$$

The third sentence says "...For a bill of less than \$10, the entire amount is due." From the context of the problem, we know that there is only a payment due of the amount of the bill is greater than or equal to \$0. This means

$$\text{If } 0 \leq x < 10, \text{ then } f(x) = x.$$

The fourth sentence says "For a bill of at least \$10 but less than \$500, the minimum due is \$10." The phrase "at least" tells us that the amount of the bill is greater than or equal to \$10, and combined with "...less than \$500" we get

$$\text{If } 10 \leq x < 500, \text{ then } f(x) = 10.$$

The first clause of the fifth sentence says "A minimum of \$30 is due on a bill of at least \$500 but less than \$1000 ..." and from this we get

$$\text{If } 500 \leq x < 1000, \text{ then } f(x) = 30.$$

The second clause of the fifth sentence says "A minimum of \$50 is due on a bill of at least \$1000 but less than \$1500 ..." and from this we get

$$\text{If } 1000 \leq x < 1500, \text{ then } f(x) = 50.$$

Finally, the last clause of the fifth sentence says "...a minimum of \$70 is due on bills of \$1500 or more" and from this we get

$$\text{If } 1500 \leq x, \text{ then } f(x) = 70.$$

Putting these pieces together, we get the piecewise function

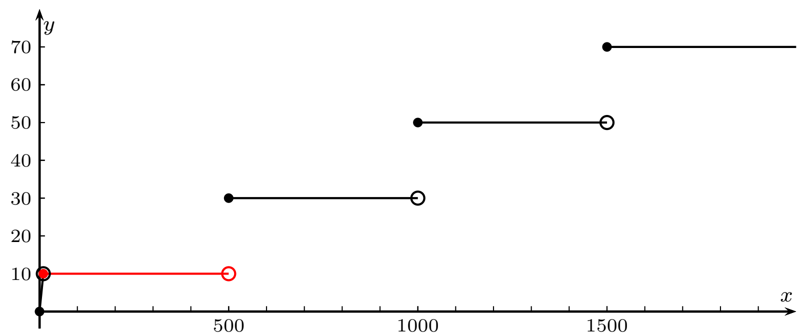
$$f(x) = \begin{cases} x, & 0 \leq x < 10 \\ 10, & 10 \leq x < 500 \\ 30, & 500 \leq x < 1000 \\ 50, & 1000 \leq x < 1500 \\ 70, & x \geq 1500 \end{cases}$$

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 102, #56.

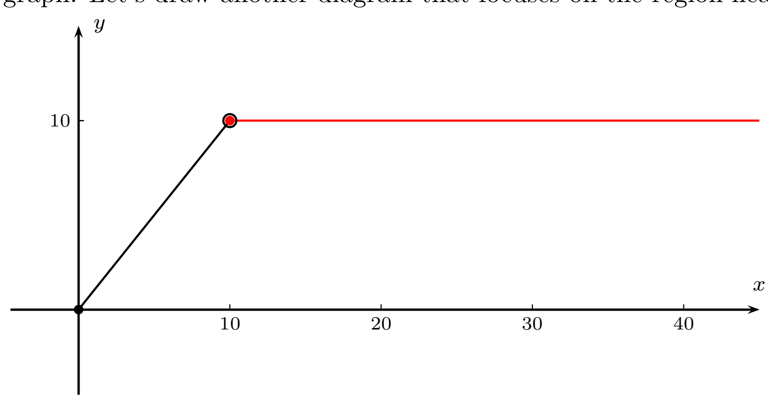
We've rewritten $1500 \leq x$ as $x \geq 1500$ to make the relationship more clear. Mathematically, they mean exactly the same thing, but most people understand $x \geq 1500$ much more easily.

Let's graph our function

$$f(x) = \begin{cases} x, & 0 \leq x < 10 \\ 10, & 10 \leq x < 500 \\ 30, & 500 \leq x < 1000 \\ 50, & 1000 \leq x < 1500 \\ 70, & x \geq 1500 \end{cases}$$



This gives a good picture of the graph for larger values of x, but near the origin it is very hard to see the graph. Let's draw another diagram that focuses on the region near the origin.



The red coloring shows that if $x = 10$, then we are using the second rule in the function, $f(x) = 10$ if $10 \leq x < 500$, to evaluate $f(10)$.

It is not uncommon to need more than one diagram to show all the features of the graph of a function. Whether we draw two or more graphs depends on the individual problem and the audience for which our work is intended.