A rectangle is inscribed in a semicircle of radius 2. See Figure 1. Let $P = (x, y)$ be the point in quadrant I that is a vertex of the rectangle and is on the circle.[1]

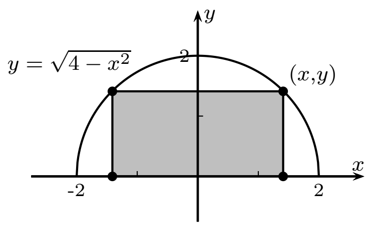


Figure 1

The statement of the problem as given includes Figure 1. On the figure, the semicircle is labeled $y = \sqrt{4 - x^2}$. Here's where that function comes from:

We know the semicircle has radius 2 and is centered at the origin. So the semicircle is part of the circle with equation $x^2 + y^2 = 4$. Let's solve for y.

$$\begin{aligned} x^2 + y^2 &= 4 \\ y^2 &= 4 - x^2 \\ \sqrt{y^2} &= \sqrt{4 - x^2} \\ |y| &= \sqrt{4 - x^2} \\ \pm y &= \sqrt{4 - x^2} \end{aligned}$$

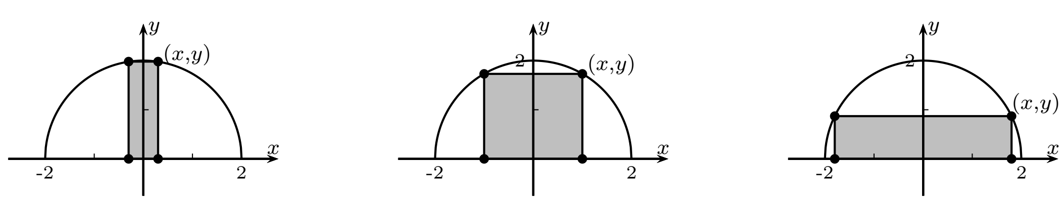
Since $\sqrt{a^2} = |a|$ and
$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$,
$\sqrt{a^2} = a$ or $-a$.
We often write this as $\sqrt{a^2} = \pm a$.

So

$$\begin{aligned} y = \sqrt{4 - x^2} &\text{ or } -y = \sqrt{4 - x^2} \\ y = \sqrt{4 - x^2} &\text{ or } y = -\sqrt{4 - x^2} \end{aligned}$$

From the given figure, we can see that the y-values are nonnegative (*i.e.* positive or zero) and thus the function is $y = \sqrt{4 - x^2}$.

Also from Figure 1, it is important to recognize that the point (x,y) in the first quadrant represents any point on the semicircle. In Figure 2, the same semicircle is shown with the inscribed rectangle drawn for three different values of x.



(a) **(b)** **(c)**

Figure 2

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 119, #8.

a. *Express the area A of the rectangle as a function of x.*

The area of a rectangle is base $\cdot$ height. From the figure, the base is $2x$, since x is the length along the x-axis from the origin to the x-intercept of the rectangle. The height is the y-value, or, in terms of x, $\sqrt{4-x^2}$.

Thus, $A(x) = 2x\sqrt{4-x^2}$.

b. *Express the perimeter p of the rectangle as a function of x.*

The perimeter of a rectangle is the sum of the lengths of its sides. Since the base is $2x$ and the height is $\sqrt{4-x^2}$, we have

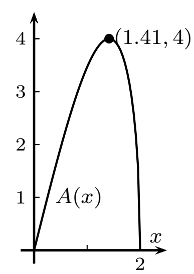
$$p(x) = 2x + \sqrt{4-x^2} + 2x + \sqrt{4-x^2}$$

So

$$p(x) = 4x + 2\sqrt{4-x^2}$$

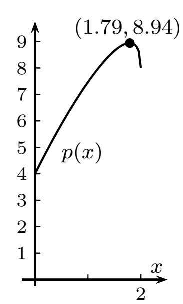
c. *Graph $A = A(x)$. For what value of x is A largest?*

Using the `calc:maximum` function on the TI-84 (or whatever graphing calculator is available), we find the local maximum of 4 occurs when $x \approx 1.41$.



d. *Graph $p = p(x)$. For what value of x is p largest?*

Using the `calc:maximum` function on the TI-84 (or whatever graphing calculator is available), we find the local maximum of ≈ 8.94 occurs when $x \approx 1.79$.



Note: Once again, we've used technology to find approximate local maximums for the area nad the perimeter. To determine the exact values of these maximums requires the concepts and techniques of calculus.