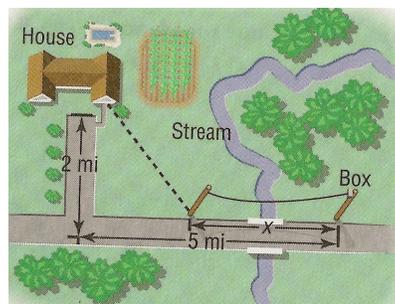
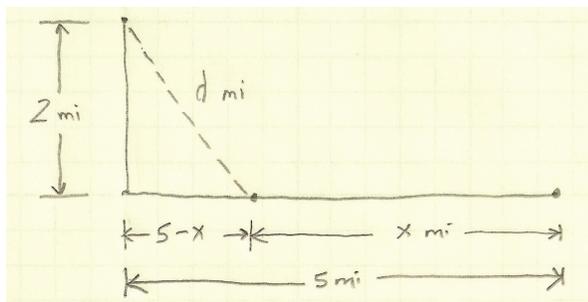


MetroMedia Cable is asked to provide service to a customer whose house is located 2 miles from the road along which the cable is buried. The nearest connection box for the cable is located 5 miles down the road.<sup>1</sup> See Figure 1a.



(a) Cute picture



(b) Math diagram

Figure 1

The book's picture is nice: house, trees, stream, connection box. But the cute picture can sometimes distract us from the important information. In Figure 1b, we've redrawn the important parts of the diagram and labeled two additional lengths. The distance from the driveway to the point where the cable installation turns off the road is  $5 - x$ , and we've named the (unknown) length of the cable off the road  $d$ .

- a. If the installation cost is \$500 per mile along the road and \$700 per mile off the road, build a model that expresses the total cost  $C$  (in dollars) of installation as a function of the distance  $x$  (in miles) from the connection box to the point where the cable installation turns off the road. Give the domain.

The length of the cable along the road is  $x$  mi and the cost per mile is 500 dollars/mile, so the cost of installing the cable along the road is  $500x$  dollars.

To find the cost of installing the cable off the road, we need an expression for  $d$ . We use the Pythagorean Theorem

$$(2)^2 + (5 - x)^2 = d^2$$

Since  $d$  is a length, we know it must be positive, so

$$d = \sqrt{4 + (5 - x)^2}$$

(Here, we are not going to multiply out  $(5 - x)^2$ , because we are going to use our calculator to evaluate our expression.) Since the length of the cable off the road is  $\sqrt{4 + (5 - x)^2}$  mi and the cost of installation off the road is 700 dollars/mile, the total cost  $C(x)$  of installing the cable as a function of  $x$  is given by

$$C(x) = 500x + 700\sqrt{4 + (5 - x)^2}$$

The domain of  $C(x)$  is the set of all allowable values of  $x$ . From the diagram, we can see that  $x$  could be 0 (the cable runs entirely off the road from the house to the connection box) or  $x$  could be 5 (the cable runs down the driveway and then entirely along the road to the connection box) or  $x$  could be any number between 0 and 5. Thus the domain of  $C(x)$  is  $[0,5]$ .

<sup>1</sup>Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 121, #22.

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b. Compute the cost if  $x = 1$  mile.

$$\begin{aligned} C(1) &= 500 \cdot 1 + 700 \cdot \sqrt{4 + (5 - 1)^2} \\ &\approx \$3630.50 \end{aligned}$$

c. Compute the cost if  $x = 3$  miles.

$$\begin{aligned} C(3) &= 500 \cdot 3 + 700 \cdot \sqrt{4 + (5 - 3)^2} \\ &\approx \$3479.90 \end{aligned}$$

d. Graph the function  $C = C(x)$ . Use TRACE to see how the cost  $C$  varies as  $x$  changes from 0 to 5.

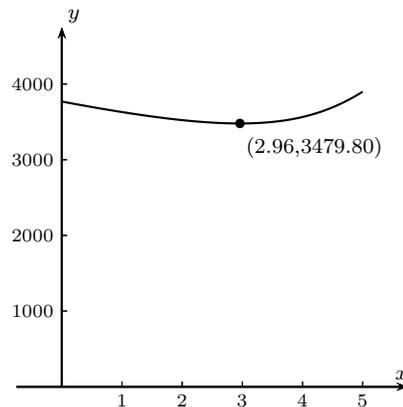


Figure 2

As  $x$  increases from 0 to 5, the values of  $C(x)$  decrease to a minimum and then increase.

e. What value of  $x$  results in the least cost?

Using the `calc:minimum` function on the TI-84 (or whatever graphing calculator is available), we find the local minimum of  $\approx \$3479.80$  occurs when  $x \approx 2.96$  mi.