Business *The price p (in dollars) and the quantity x sold of a certain product obey the demand equation*[1]

$$x = -20p + 500 \qquad 0 < p \leq 25$$

a. *Express the revenue R as a function of x.*

Recall that *revenue* = *price* · *quantity*. Because we want the revenue as a function of x, we need to write both the price and the quantity in terms of x.

The quantity is simple; it is x.

To write the price in terms of x, we need to solve the given demand equation for p in terms of x.

$$\begin{aligned} x &= -20p + 500 \\ 20p &= -x + 500 \\ \frac{1}{20} \cdot 20p &= \frac{1}{20}(-x + 500) \\ p &= -\frac{1}{20}x + 25 \end{aligned}$$

The revenue in terms of x is given by

$$\begin{aligned} R &= x\left(-\frac{1}{20}x + 25\right) \\ &= -\frac{1}{20}x^2 + 25x \end{aligned}$$

Thus

$$R(x) = -\frac{1}{20}x^2 + 25x$$

b. *What is the revenue if 20 units are sold?*

$$\begin{aligned} R(x) &= -\tfrac{1}{20}x^2 + 25x \\ R(20) &= -\tfrac{1}{20}(20)^2 + 25(20) \\ R(20) &= 480 \end{aligned}$$

Thus, if we produce and sell 20 units, the revenue will be \$480.

c. *What quantity x maximizes revenue? What is the maximum revenue?*

Note that the revenue is a quadratic function

$$R(x) = -\tfrac{1}{20}x^2 + 25x$$

with $a = -1/20$, $b = 25$, and $c = 0$, so the axis of symmetry is

$$x = -\frac{b}{2a} = -\frac{25}{2 \cdot -1/20} = 250$$

The quadratic coefficient is $-1/20$, so the parabola opens downward and the revenue has a maximum. The maximum revenue is given by

$$R(250) = -\frac{1}{20}(250)^2 + 25(250) = 3125$$

Thus the maximum revenue of \$3125 occurs when we produce and sell 250 units.

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 165, #6.

d. *What price should the company charge to maximize revenue?*

In part (a), we expressed the price p as a function of the quantity x. We'll use that relationship to find the price, along with our result from part (b).

$$p = -\frac{1}{20}x + 25$$
$$p = -\frac{1}{20} \cdot 250 + 25$$
$$p = 12.5$$

Thus, the price should be \$12.50 to maximize revenue.

e. *What price should the company charge to earn at least \$3000 in revenue?*

Here, we want our revenue to be greater than or equal to \$3000. We'll solve this graphically.

First, we graph our revenue function. See Figure 1a.

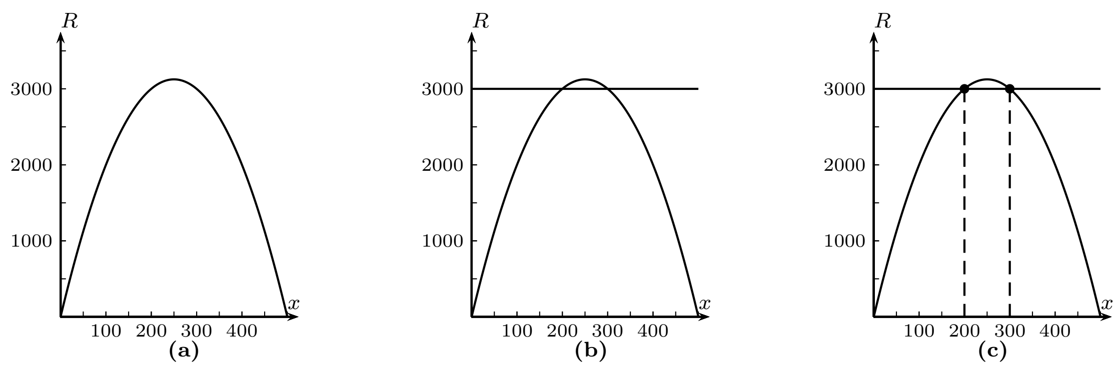

Figure 1

Now we'll graph the desired revenue, $R = 3000$. See Figure 1b. We want the prices for which revenue is "at least" \$3000, meaning we want the prices for which the revenue is greater than or equal to \$3000. Our graphs show the relationship between quantity x and revenue, so using the `calc:intersect` function on the TI-84 (or whatever graphing calculator is available), we find intersections at the quantities $x = 200$ and $x = 300$. See Figure 1c.

Finally, using the price as a function of quantity equation from part (**d**), we find

if $x = 200$, then

$$p = -\frac{1}{20}x + 25$$
$$p = -\frac{1}{20} \cdot 200 + 25$$
$$p = 15$$

if $x = 300$, then

$$p = -\frac{1}{20}x + 25$$
$$p = -\frac{1}{20} \cdot 300 + 25$$
$$p = 10$$

so if we charge a price from \$10 to \$15, inclusive, then the revenue will be at least \$3000.