

Precalculus, Section 4.1, #90
 Polynomial Functions and Models

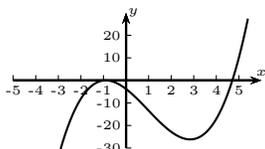
Analyze the polynomial function by following Steps 1 through 8 on page 192.¹

$$f(x) = x^3 - 2.91x^2 - 7.668x - 3.8151$$

Step 1 Determine the end behavior of the graph of the function.

The leading term x^3 has odd degree and a positive coefficient of 1. The graph of f behaves like $y = x^3$. Another way of expressing this is as $x \rightarrow \infty, f(x) \rightarrow \infty$ and as $x \rightarrow -\infty, f(x) \rightarrow -\infty$.

Step 2 Graph the function using a graphing utility.



Step 3 Use a graphing utility to approximate the x - and y -intercepts of the graph.

Using the `calc:zero` function on the TI-84 (or whatever graphing calculator is available), we find a zero at $x \approx -0.90$ and another at $x \approx 4.71$, so the x -intercepts are $(-0.90, 0)$ and $(4.71, 0)$.

To find the y -intercept(s), we substitute $x = 0$:

$$f(x) = x^3 - 2.91 * x^2 - 7.668 * x - 3.8151$$

$$f(0) = 0^3 - 2.91 * 0^2 - 7.668 * 0 - 3.8151$$

$$f(0) = -3.8151$$

thus the y -intercept is the point $(0, -3.8151)$.

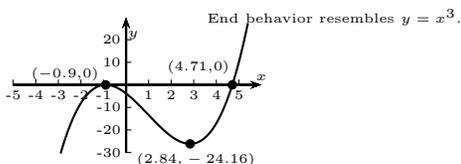
Step 4 Use a graphing utility to create a TABLE to find points on the graph around each x -intercept.

x	$f(x)$
-1	-0.0571
-0.8	-0.0551
4	-17.0471
5	10.0949

Step 5 Approximate the turning points of the graph.

Using the `calc:minimum` function on the TI-84 (or whatever graphing calculator is available), we find a turning point occurs at $x \approx 2.84$, and from the graph and table, we find a turning point at $x \approx -0.90$.

Step 6 Use the information in Steps 1 through 5 to draw a complete graph of the function by hand.



Step 7 Find the domain and range of the function.

Because the function is a polynomial function, we know the domain is all real numbers, *i.e.*, $(-\infty, \infty)$. Because the end behavior is like $y = x^3$, the range is also all real numbers, *i.e.*, $(-\infty, \infty)$.

Step 8 Use the graph to determine where the function is increasing and where it is decreasing.

Using our work from Step 5 and the graph from Step 6, we can see that the function is increasing on $(-\infty, -0.9)$, decreasing on $(-0.9, 2.84)$, and increasing on $(2.84, \infty)$.

¹Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 195, #90.