Use the Intermediate Value Theorem to show the function has a zero in the given interval. Approximate the zero rounded to two decimal places.[1]

$$f(x) = 3x^3 - 10x + 9; \quad [-3, -2]$$

Let's examine The Intermediate Value Theorem (IVT):

The Intermediate Value Theorem

Let f denote a continuous function. If $a < b$ and if $f(a)$ and $f(b)$ are of opposite sign, then f has at least one zero between a and b.

The hypothesis of the IVT (the "If..." part) includes three conditions:

i ... f denote a continuous function ...

ii ... $a < b$ ...

iii ... $f(a)$ and $f(b)$ are of opposite sign ...

If all three of these conditions are true, then the conclusion "... then f has at least one zero between a and b." must also be true.

i Our function $f(x) = 3x^3 - 10x + 9$ is a polynomial, and all polynomials are continuous for all real numbers.

ii The given interval is $[-3, -2]$ and $-3 < -2$.

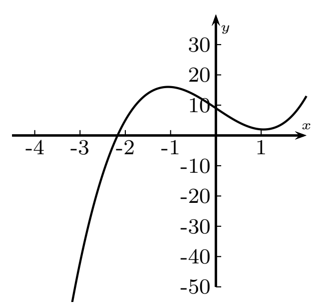
iii

$$\begin{aligned} f(-3) &= 3(-3)^3 - 10(-3) + 9 \\ &= -42 \end{aligned} \qquad \begin{aligned} f(-2) &= 3(-2)^3 - 10(-2) + 9 \\ &= 5 \end{aligned}$$

so $f(-3)$ and $f(-2)$ are of opposite sign.

All three conditions of the hypothesis are satisfied, and thus the function $f(x) = 3x^3 - 10x + 9$ must have a zero in the interval $[-3, -2]$.

The graph of our function $f(x) = 3x^3 - 10x + 9$ is shown at right. Using the `calc:zero` function on the TI-84 (or whatever graphing calculator is available), we find the zero at $x \approx -2.17$.



[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 210, #76.