Form a polynomial function $f(x)$ with real coefficients having the given degree and zeros. Answers will vary depending on the choice of the leading coefficient. Use a graphing utility to graph the functions and verify the result.[1]

Degree 6; zeros: $i, 4-i, 2+i$

We want a polynomial function $f(x)$ with real coefficients. From the Conjugate Pairs Theorem, we know if $r = a + bi$ is a zero of f, then the complex conjugate $\bar{r} = a - bi$ is also a zero. So the zeros of our desired function are

$$i, -i, 4-i, 4+i, 2+i, 2-i$$

From the Fundamental Theorem of Algebra, we know that for each zero r_i there is a corresponding linear factor of the form $(x - r_i)$. Our function can be written as

$$f(x) = a\,(x-(i))\,(x-(-i))\,(x-(4-i))\,(x-(4+i))\,(x-(2+i))\,(x-(2-i))$$

where a can be any real number, $a \neq 0$. Let's simplify

$$\begin{aligned}
&= a\,(x-i)\,(x+i)\,(x-4+i)\,(x-4-i)\,(x-2-i)\,(x-2+i) \\
&= a\,((x-i)\,(x+i))\,((x-4+i)\,(x-4-i))\,((x-2-i)\,(x-2+i)) \\
&= a\left(x^2+xi-xi-i^2\right)\left(x^2-4x-xi-4x+16+4i+xi-4i-i^2\right)\cdot \\
&\qquad\qquad \left(x^2-2x+xi-2x+4-2i-xi+2i-i^2\right) \\
&= a\left(x^2-(-1)\right)\left(x^2-8x+16-(-1)\right)\left(x^2-4x+4-(-1)\right) \\
&= a\left(x^2+1\right)\left(x^2-8x+17\right)\left(x^2-4x+5\right) \\
&= a\left(x^4-8x^3+17x^2+x^2-8x+17\right)\left(x^2-4x+5\right) \\
&= a\left(x^4-8x^3+18x^2-8x+17\right)\left(x^2-4x+5\right) \\
&= a\left(x^6-4x^5+5x^4-8x^5+32x^4-40x^3+18x^4-72x^3+90x^2-8x^3+32x^2-40x+17x^2-68x+85\right) \\
&= a\left(x^6-12x^5+55x^4-120x^3+139x^2-108x+85\right)
\end{aligned}$$

Now a can be any real number, so let's keep things simple and choose $a = 1$. Thus, our function is

$$f(x) = x^6 - 12x^5 + 55x^4 - 120x^3 + 139x^2 - 108x + 85$$

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 215, #20.