Precalculus, Section 5.2, #56
One-to-One Functions; Inverse Functions

The function f is one-to-one. Find its inverse and check your answer. Graph f, f^{-1}, and $y = x$ on the same coordinate axes.[1]

$$f(x) = x^2 + 9 \qquad x \geq 0$$

We begin by writing

$$y = x^2 + 9$$

Now we solve for x

$$\begin{aligned} y - 9 &= x^2 \\ \sqrt{y-9} &= \sqrt{x^2} \end{aligned}$$

Since $\sqrt{a^2} = |a|$ and
$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$,
$\sqrt{a^2} = a$ or $-a$.
We often write this as $\sqrt{a^2} = \pm a$.

so

$$x = \sqrt{y-9} \qquad \text{or} \qquad x = -\sqrt{y-9}$$

but we are given that $x \geq 0$, so we choose the function where x is nonnegative. Thus,

$$x = \sqrt{y-9}$$

and we rewrite this as

$$f^{-1}(x) = \sqrt{x-9}, \qquad x \geq 9$$

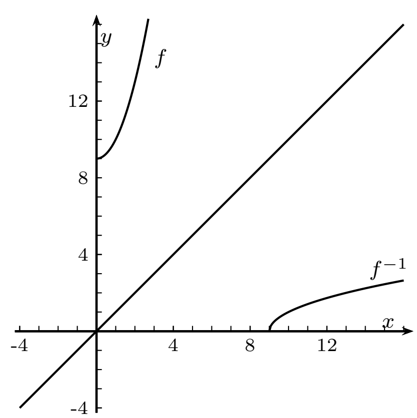
We can check our work by computing $f\big(f^{-1}(x)\big)$ and $f^{-1}(f(x))$. They should both equal x.

$$\begin{aligned} f\big(f^{-1}(x)\big) &= \big(\sqrt{x-9}\big)^2 + 9 \\ &= x - 9 + 9 \\ &= x \end{aligned}$$

and

$$\begin{aligned} f^{-1}(f(x)) &= \sqrt{x^2 + 9 - 9} \\ &= \sqrt{x^2} \\ &= x, \quad \text{since we know } x \geq 0 \end{aligned}$$

The graph is shown at right.



[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 269, #56.