

Learning Curve Psychologists sometimes use the function

$$L(t) = A(1 - e^{-kt})$$

to measure the amount L learned at time t . The number A represents the amount to be learned, and the number k measures the rate of learning. Suppose that a student has an amount A of 200 vocabulary words to learn. A psychologist determines that the student learned 20 vocabulary words after 5 minutes.¹

- (a) Determine the rate of learning k .

The problem description tells us $A = 200$, $t = 5$, and $L = 20$. Substituting these into the given function, we get

$$\begin{aligned} L(t) &= A(1 - e^{-kt}) \\ 20 &= 200(1 - e^{-k \cdot 5}) \end{aligned}$$

Now we solve for k

$$\begin{aligned} \frac{1}{200} \cdot 20 &= \frac{1}{200} \cdot 200(1 - e^{-k \cdot 5}) \\ \frac{1}{10} &= 1 - e^{-5k} \\ \frac{1}{10} - 1 &= 1 - e^{-5k} - 1 \\ -\frac{9}{10} &= -e^{-5k} \\ \frac{9}{10} &= e^{-5k} \\ \ln\left(\frac{9}{10}\right) &= \ln(e^{-5k}) \\ \ln\left(\frac{9}{10}\right) &= -5k \\ -\frac{1}{5} \cdot \ln(0.9) &= -\frac{1}{5} \cdot -5k \\ -\frac{\ln(0.9)}{5} &= k \\ 0.0211 &\approx k \end{aligned}$$

- (b) Approximately how many words will the student have learned after 10 minutes?

Substituting our result from part (a) into the function gives us

$$L(t) = 200(1 - e^{-0.0211t})$$

Then substituting $t = 10$ we get

$$\begin{aligned} L(10) &= 200(1 - e^{-0.0211 \cdot 10}) \\ &\approx 38 \end{aligned}$$

So the student will learn about 38 words in 10 minutes of study.

¹Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 300, #126.

Precalculus
Logarithmic Functions

(c) *After 15 minutes?*

Substituting $t = 15$ we get

$$\begin{aligned}L(15) &= 200(1 - e^{-0.0211 \cdot 15}) \\ &\approx 54\end{aligned}$$

So the student will learn about 54 words in 15 minutes of study.

(d) *How long does it take for the student to learn 180 words?*

We are given that $L = 180$. Substituting

$$\begin{aligned}180 &= 200(1 - e^{-0.0211t}) \\ \frac{180}{200} &= 1 - e^{-0.0211t} \\ \frac{9}{10} - 1 &= 1 - e^{-0.0211t} - 1 \\ -\frac{1}{10} &= -e^{-0.0211t} \\ 0.1 &= e^{-0.0211t} \\ \ln(0.1) &= \ln(e^{-0.0211t}) \\ \ln(0.1) &= -0.0211t \\ \frac{\ln(0.1)}{-0.0211} &= t \\ 109.13 &\approx t\end{aligned}$$

Thus it will take the student about 109 minutes to learn 180 words.