Decomposition of Dinitrogen Pentoxide *At 45° C, dinitrogen pentoxide (N_2O_5) decomposes into nitrous dioxide (NO_2) and oxygen (O_2) according to the law of uninhibitted decay. An initial amount of 0.25 mole of dinitrogen pentoxide decomposes to 0.15 mole in 17 minutes. How much dinitrogen pentoxide will remain after 30 minutes? How long will it take until 0.01 mole of dinitrogen pentoxide remains?*[1]

The law of uninhibited decay is

$$A(t) = A_0 \mathrm{e}^{kt}$$

so we need to determine the values of A_0 and k for the model, and then we can respond to the questions.

The problem states that the initial amount of $\mathrm{N_2O_5}$ is 0.25 mole; this is the value of A_0. Substituting,

$$A(t) = 0.25\mathrm{e}^{kt}$$

Now we'll substitute the 0.15 mole in 17 minutes, and solve for k

$$\begin{aligned}
0.15 &= 0.25\mathrm{e}^{k\cdot 17} \\
\frac{0.15}{0.25} &= \mathrm{e}^{17k} \\
0.6 &= \mathrm{e}^{17k} \\
\ln(0.6) &= \ln\left(\mathrm{e}^{17k}\right) \\
\ln(0.6) &= 17k \\
\frac{\ln(0.6)}{17} &= k
\end{aligned}$$

or

$$k \approx -0.0300$$

Thus the mathematical model for the decay of 0.25 moles of dinitrogen pentoxide is

$$A(t) = 0.25\mathrm{e}^{-0.0300t}$$

After 30 minutes, there will be

$$\begin{aligned}
A(30) &= 0.25\mathrm{e}^{-0.0300\cdot 30} \\
&\approx 0.1016 \text{ moles}
\end{aligned}$$

To find how long until only 0.01 moles remain, we substitute and solve

$$\begin{aligned}
0.01 &= 0.25\mathrm{e}^{-0.0300t} \\
\frac{0.01}{0.25} &= \mathrm{e}^{-0.0300t} \\
0.04 &= \mathrm{e}^{-0.0300t} \\
\ln(0.04) &= \ln\left(\mathrm{e}^{-0.0300t}\right) \\
\ln(0.04) &= -0.0300t \\
\frac{\ln(0.04)}{-0.0300} &= t
\end{aligned}$$

so

$$t \approx 107 \text{ minutes}$$

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 336, #18.