Find the exact value of each of the remaining trigonometric functions of θ.[1]

$$\sec(\theta) = -2 \qquad \tan(\theta) > 0$$

Since $\sec(\theta) = \dfrac{1}{\cos(\theta)}$ and $\sec(\theta) < 0$, we know $\cos(\theta) < 0$. This tells us that θ must be in quadrant II or quadrant III. But we are given that $\tan(\theta) > 0$, so θ must be in quadrant III.

Now $\sec(\theta) = -2 = \dfrac{-2}{1}$, so we know $\cos(\theta) = \dfrac{1}{\frac{-2}{1}} = \dfrac{1}{-2} = \dfrac{-1}{2}$. Thus we can take $x = -1$ and $r = 2$.

To find y we use

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ 2 &= \sqrt{(-1)^2 + y^2} \\ 2 &= \sqrt{1 + y^2} \\ 4 &= 1 + y^2 \\ 3 &= y^2 \end{aligned}$$

so

$$y = \sqrt{3} \quad \text{or} \quad y = -\sqrt{3}$$

but we know $y < 0$ since θ is in quadrant III, so

$$y = -\sqrt{3}$$

Knowing values for x, y, and r, we can now write the other trig functions.

$$\sec(\theta) = -2 \qquad \tan(\theta) = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

$$\cos(\theta) = \frac{-1}{2} = -\frac{1}{2} \qquad \sin(\theta) = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$$

$$\csc(\theta) = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3} \qquad \cot(\theta) = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}$$

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 395, #58.