

Precalculus, Section 6.4, #58
 Graphs of the Sine and Cosine Functions

Graph the function. Be sure to label key points and show a least two cycles. Use the graph to determine the domain and range of the function.¹

$$y = -\frac{1}{2} \sin\left(\frac{\pi}{8}x\right) + \frac{3}{2}$$

The period of our function is $\frac{2\pi}{\frac{\pi}{8}} = 16$. The subintervals will have length $16/4 = 4$ and begin at $x = 0$ since there is no horizontal translation (phase shift). Thus the key points occur at $x = 0, 4, 8, 12, 16$. We compute

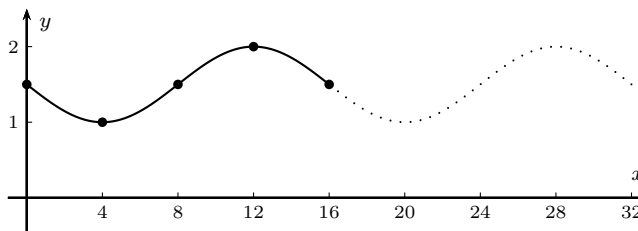
$$\begin{aligned} y &= -\frac{1}{2} \sin\left(\frac{\pi}{8}x\right) + \frac{3}{2} \\ y(0) &= -\frac{1}{2} \sin\left(\frac{\pi}{8} \cdot 0\right) + \frac{3}{2} \\ &= -\frac{1}{2} \sin(0) + \frac{3}{2} \\ &= -\frac{1}{2} \cdot 0 + \frac{3}{2} \\ &= \frac{3}{2} \end{aligned}$$

$$\begin{aligned} y &= -\frac{1}{2} \sin\left(\frac{\pi}{8}x\right) + \frac{3}{2} \\ y(12) &= -\frac{1}{2} \sin\left(\frac{\pi}{8} \cdot 12\right) + \frac{3}{2} \\ &= -\frac{1}{2} \sin\left(\frac{3\pi}{2}\right) + \frac{3}{2} \\ &= -\frac{1}{2} \cdot -1 + \frac{3}{2} \\ &= 2 \end{aligned}$$

$$\begin{aligned} y &= -\frac{1}{2} \sin\left(\frac{\pi}{8}x\right) + \frac{3}{2} \\ y(4) &= -\frac{1}{2} \sin\left(\frac{\pi}{8} \cdot 4\right) + \frac{3}{2} \\ &= -\frac{1}{2} \sin\left(\frac{\pi}{2}\right) + \frac{3}{2} \\ &= -\frac{1}{2} \cdot 1 + \frac{3}{2} \\ &= 1 \end{aligned}$$

$$\begin{aligned} y &= -\frac{1}{2} \sin\left(\frac{\pi}{8}x\right) + \frac{3}{2} \\ y(16) &= -\frac{1}{2} \sin\left(\frac{\pi}{8} \cdot 16\right) + \frac{3}{2} \\ &= -\frac{1}{2} \sin(2\pi) + \frac{3}{2} \\ &= -\frac{1}{2} \cdot 0 + \frac{3}{2} \\ &= \frac{3}{2} \end{aligned}$$

$$\begin{aligned} y &= -\frac{1}{2} \sin\left(\frac{\pi}{8}x\right) + \frac{3}{2} \\ y(8) &= -\frac{1}{2} \sin\left(\frac{\pi}{8} \cdot 8\right) + \frac{3}{2} \\ &= -\frac{1}{2} \sin(\pi) + \frac{3}{2} \\ &= -\frac{1}{2} \cdot 0 + \frac{3}{2} \\ &= \frac{3}{2} \end{aligned}$$



The graph is shown at right.

The domain is all real numbers $(-\infty, \infty)$ and the range is $[1, 2]$.

¹Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 410, #58.