***Hours of Daylight*** *According to Old Farmer's Almanac, in Detroit, Michigan, the number of hours of sunllight on the summer solstice of 2011 was 15.30, and the number of hours of sunlight on the winter solstice was 9.10.*[1]

(a) *Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.*

amplitude $= \dfrac{15.30 - 9.10}{2} = 3.1$

vertical shift $= \dfrac{15.30 + 9.10}{2} = 12.2$

The period is 365 days, so

$$365 = \frac{2\pi}{\omega} \quad \text{or} \quad \omega = \frac{2\pi}{365}$$

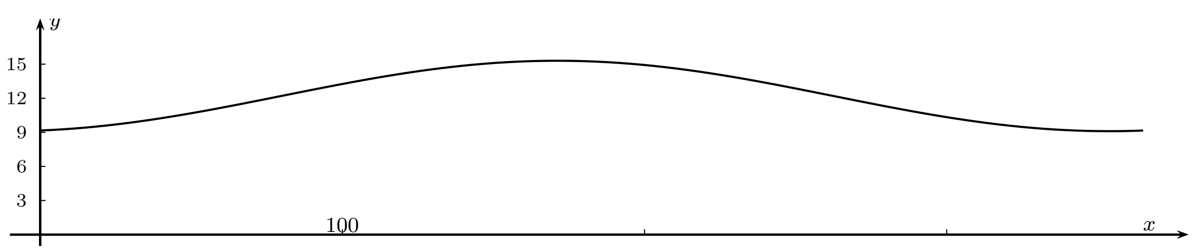
Now we need to find the phase shift, $\phi$. The period is 365 days. If we divide that into 4 subintervals, each subinterval will be $360/4 = 91.25$ days long. So our subintervals will be [0,91.25], $[91.25, 182.5]$, $[182.5, 273.75]$, and $[273.75, 365]$. From our book, we know that the summer solstice (longest day of the year) occurs on day 171. From our knowledge of the sine curve, there is a maximum at the end of the first subinterval. So we must shift the graph $171 - 91.25 = 79.75$ days to the right. We do this by using $(x - 79.75)$ inside the sine function.

Thus we have $y = 3.1\sin\left(\dfrac{2\pi}{365}(x - 79.75)\right) + 12.2$ or $y = 3.1\sin(0.0172x - 1.3728) + 12.2$

(b) *Use the function found in part (a) to predict the hours of daylight on April 1st, the 91st day of the year.*

$$y = 3.1\sin\left(\frac{2\pi}{365}(x - 79.75)\right) + 12.2$$
$$y = 3.1\sin\left(\frac{2\pi}{365}(91 - 79.75)\right) + 12.2$$
$$y \approx 12.80 \text{ hours}$$

(c) *Draw a graph of the function found in part (a).*




(d) *Look up the number of hours of sunlight for April 1 in the Olfd Farmer's almanac, and compare the actual hours of daylight to the results found in part (b).*

Wolfram-Alpha indicate 12 hours and 43 minutes of daylight for April 1 in Detroit Michigan. Our function gave 12.80 hours, or 12 hours and 48 minutes. The mathematical model gives a very close approximation.

[1] Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 431, #36.