

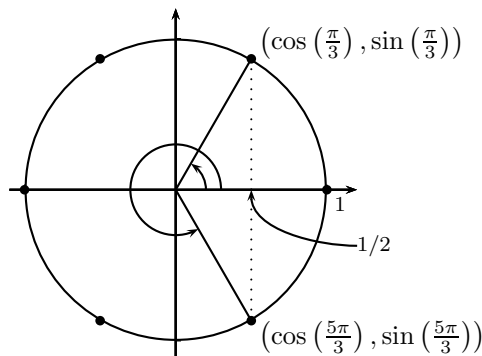
Precalculus, Section 7.3, #34
 Trigonometric Equations

Solve the equation on the interval $0 \leq \theta \leq 2\pi$.¹

$$\cos\left(\frac{\theta}{3} - \frac{\pi}{4}\right) = \frac{1}{2}$$

From our knowledge of the unit circle, we know the input to the cosine, $\frac{\theta}{3} - \frac{\pi}{4}$, must be either $\frac{\pi}{3}$ or $\frac{5\pi}{3}$, since $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ and $\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$. So we solve

$$\begin{aligned} \frac{\theta}{3} - \frac{\pi}{4} &= \frac{\pi}{3} & \text{or} & & \frac{\theta}{3} - \frac{\pi}{4} &= \frac{5\pi}{3} \\ \frac{\theta}{3} &= \frac{\pi}{3} + \frac{\pi}{4} & & & \frac{\theta}{3} &= \frac{\pi}{4} + \frac{5\pi}{3} \\ \frac{\theta}{3} &= \frac{7\pi}{12} & & & \frac{\theta}{3} &= \frac{23\pi}{12} \\ \frac{\theta}{3} \cdot 3 &= \frac{7\pi}{12} \cdot 3 & & & \frac{\theta}{3} \cdot 3 &= \frac{23\pi}{12} \cdot 3 \\ \theta &= \frac{7\pi}{4} & \text{or} & & \theta &= \frac{23\pi}{4} \end{aligned}$$



Note that $\frac{23\pi}{4} > 2\pi$ and so is not a solution on the given interval.

Thus, the solution to the equation on the interval $0 \leq \theta \leq 2\pi$ is $\theta = \frac{7\pi}{4}$.

¹Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 465, #34.