

Precalculus, Section 7.4, #48
Trigonometric Identities

Establish the identity¹

$$\frac{\cos(\theta) + 1}{\cos(\theta) - 1} = \frac{1 + \sec(\theta)}{1 - \sec(\theta)}$$

Let's start with the right side, because it involves more complicated trig functions. our goal is to make the right side look like the left side.

$$\begin{aligned} \frac{1 + \sec(\theta)}{1 - \sec(\theta)} &= \frac{1 + \frac{1}{\cos(\theta)}}{1 - \frac{1}{\cos(\theta)}} \\ &= \frac{\frac{\cos(\theta)}{\cos(\theta)} + \frac{1}{\cos(\theta)}}{\frac{\cos(\theta)}{\cos(\theta)} - \frac{1}{\cos(\theta)}} \\ &= \frac{\frac{\cos(\theta) + 1}{\cos(\theta)}}{\frac{\cos(\theta) - 1}{\cos(\theta)}} \\ &= \frac{\cos(\theta) + 1}{\cos(\theta)} \cdot \frac{\cos(\theta)}{\cos(\theta) - 1} \\ &= \frac{\cos(\theta) + 1}{\cos(\theta) - 1} \end{aligned}$$

Thus, $\frac{1 + \sec(\theta)}{1 - \sec(\theta)} = \frac{\cos(\theta) + 1}{\cos(\theta) - 1}$ or, as it was given

$$\frac{\cos(\theta) + 1}{\cos(\theta) - 1} = \frac{1 + \sec(\theta)}{1 - \sec(\theta)}$$

¹Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 474, #48.