

Precalculus, Section 7.5, #62  
Sum and Difference Formulas

---

Establish the identity<sup>1</sup>

$$\frac{\cos(\alpha - \beta)}{\sin(\alpha)\cos(\beta)} = \cot(\alpha) + \tan(\beta)$$

Since this identity includes the expression  $\cos(\alpha - \beta)$ , we'll probably need to recall

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

Let's start with the left side and try to transform it into the right side.

$$\begin{aligned}\frac{\cos(\alpha - \beta)}{\sin(\alpha)\cos(\beta)} &= \frac{\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)}{\sin(\alpha)\cos(\beta)} \\ &= \frac{\cos(\alpha)\cos(\beta)}{\sin(\alpha)\cos(\beta)} + \frac{\sin(\alpha)\sin(\beta)}{\sin(\alpha)\cos(\beta)} \\ &= \frac{\cos(\alpha)}{\sin(\alpha)} + \frac{\sin(\beta)}{\cos(\beta)} \\ &= \cot(\alpha) + \tan(\beta)\end{aligned}$$

Now let's try to transform the right side into the left side.

$$\begin{aligned}\cot(\alpha) + \tan(\beta) &= \frac{\cos(\alpha)}{\sin(\alpha)} + \frac{\sin(\beta)}{\cos(\beta)} \\ &= \frac{\cos(\alpha)}{\sin(\alpha)} \cdot \frac{\cos(\beta)}{\cos(\beta)} + \frac{\sin(\beta)}{\cos(\beta)} \cdot \frac{\sin(\alpha)}{\sin(\alpha)} \\ &= \frac{\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)}{\sin(\alpha)\cos(\beta)} \\ &= \frac{\cos(\alpha - \beta)}{\sin(\alpha)\cos(\beta)}\end{aligned}$$

---

<sup>1</sup>Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 486, #22.