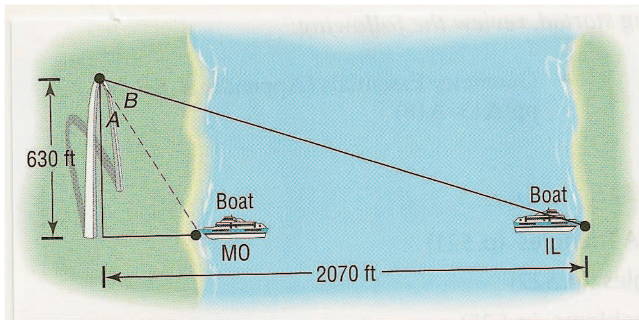
Estimating the Width of the Mississippi River *A tourist at the top of the Gateway Arch (height 630 feet) in St. Louis, Missouri, observes a boat moored on the Illinois side of the Mississippi River 2070 feet directly across from the Arch. She also observes a boat moored on the Missouri side directly across from the first boat (see diagram). Given that* $B = \cot^{-1}\left(\frac{67}{55}\right)$, *estimate the width of the Mississippi River at the St. Louis riverfront.*[1]



To find the distance between the boats, we need the distance from the Gateway Arch to the boat on the MO side. To find that distance, we need the angle at A so we can use the tangent function.

From the big right triangle, we get $\cot(\angle IL) = \frac{2070}{630}$ so $\angle IL = \cot^{-1}\left(\frac{23}{7}\right)$. Again from the big right triangle,

$$\angle A + \cot^{-1}\left(\frac{67}{55}\right) + \cot^{-1}\left(\frac{23}{7}\right) = \frac{\pi}{2}$$

$$\angle A = \frac{\pi}{2} - \cot^{-1}\left(\frac{67}{55}\right) - \cot^{-1}\left(\frac{23}{7}\right)$$

$$\angle A \approx 0.5880$$

$$\approx 33.67^\circ$$

From the smaller right triangle, we get

$$\tan(33.67) = \frac{x}{630}$$

$$x = 630 \cdot \tan(33.67)$$

$$\approx 419.68$$

So the distance from MO to IL is $2070 - 149.68 = 1650.32$ or about 1650 feet.

[1]Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 519, #74.