The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May it cost her $380 to drive 480 mi and in June it cost her $460 to drive 800 mi.[1]

(a) *Express the monthly cost C as a function of the distance driven d, assuming that a linear relationship gives a suitable model.*

The phrase "…cost of driving a car depends on the number of miles …" tells us the input to the function will be d, the number of miles driven and the output will be $C(d)$, the monthly cost, in dollars. This same conclusion can be reached from decoding the phrase "…cost C as a function of the distance driven d …". Thus we have the points (480,380) and (800,460). (Note that these points are of the form (input, output).)

We are told to use a linear relationship, so we need the slope and vertical intercept to write the function rule.

$$\begin{aligned} m &= \frac{460-380}{800-480} \\ &= \frac{80}{320} \\ &= \frac{1}{4} \end{aligned}$$

At this point, our linear function is

$$C(d) = \frac{1}{4}d + b$$

To find the vertical intercept b, we substitute either point into the function and solve for b.

$$\begin{aligned} C(d) &= \frac{1}{4}d + b \\ 380 &= \frac{1}{4} \cdot 480 + b \\ 380 &= 120 + b \\ 260 &= b \end{aligned}$$

So the function is

$$C(d) = \frac{1}{4}d + 260$$

(b) *Use part (a) to predict the cost of driving 1500 miles per month.*

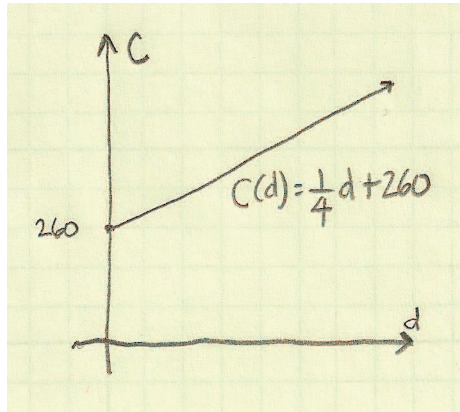
$$\begin{aligned} C(d) &= \frac{1}{4}d + 320 \\ C(1500) &= \frac{1}{4} \cdot 1500 + 260 \\ C(1500) &= 635 \end{aligned}$$

Thus the cost of driving 1500 miles per month is $635.

[1] Stewart, *Calculus, Early Transcendentals*, p. 34, #20.

(c) *Draw the graph of the linear function. What does the slope represent?*

We have made a sketch graph of the function at right. This sketch shows all of the important features of the function: vertical intercept, positive slope, axes labels, and the function is clearly labeled.



The slope $\frac{1}{4}$ has units $\frac{\text{dollars}}{\text{mile}}$, and thus represents a cost of \$1 to drive 4 additional miles. We can also express this as \$0.25 per mile. (Please note that these are *additional* miles because the slope term, $\frac{1}{4}d$, only includes the variable cost of driving.)

(d) *What does the C-intercept represent?*

The C-intercept represents the monthly cost when we drive 0 miles. It is the fixed costs of owning a car, including payment, maintenance, and insurance. For this specific function, the monthly fixed cost is \$260.

(e) *Why does a linear function give a suitable model in this situation?*

If we assume a set price for fuel, it is reasonable to expect, for example, that the cost to drive the 25th mile of the month will be the same as the cost to drive the 273rd mile of the month. This tells us that the slope is constant. Thus a linear model is the most suitable.