A bacteria culture starts with 500 bacteria and doubles in size every half hour.[1]

(a) *How many bacteria are there after 3 hours?*

We are told "...doubles in size every half hour." Let's make a table of the time and population:

t	0	0.5	1.0	1.5	2.0	2.5	3.0
bacteria	500	1000	2000	4000	8000	16000	32000

Thus, after three hours, the population of bacteria is 32,000.

(b) *How many bacteria are there after t hours?*

In t hours, there are $2t$ doubling periods. (For example, after 4 hours, the population has doubled 8 times.) The initial value is 500, so the population P at time t is given by

$$P(t) = 500 \cdot 2^{2t}$$

This is an acceptable response, but in calculus and all advanced mathematics and science, we will almost always want to use the natural exponential base, e. Let's redo the problem using the natural exponential growth function

$$P(t) = P_0 \mathrm{e}^{rt}$$

We are given that the initial population is 500 bacteria. So $P_0 = 500$ and we have

$$P(t) = 500\mathrm{e}^{rt}$$

We know that after 1 hour, there are 2000 bacteria. (We could've used any other pair from our table that we wish.) Substituting into our function, we get

$$\begin{aligned} P(t) &= 500\mathrm{e}^{rt} \\ 2000 &= 500\mathrm{e}^{r\cdot 1} \\ 2000 &= 500\mathrm{e}^{r} \\ \frac{1}{500} \cdot 2000 &= \frac{1}{500} \cdot 500\mathrm{e}^{r} \\ 4 &= \mathrm{e}^{r} \end{aligned}$$

and now we use the natural logarithm to solve for r

$$\begin{aligned} \ln(4) &= \ln(\mathrm{e}^{r}) \\ \ln(4) &= r \cdot \ln(\mathrm{e}) \\ \ln(4) &= r \\ 1.3863 &\approx r \end{aligned}$$

Thus the function $P(t) = 500\mathrm{e}^{1.3863t}$ gives the number of bacteria after t hours.

[1] Stewart, *Calculus, Early Transcendentals*, p. 54, #30.

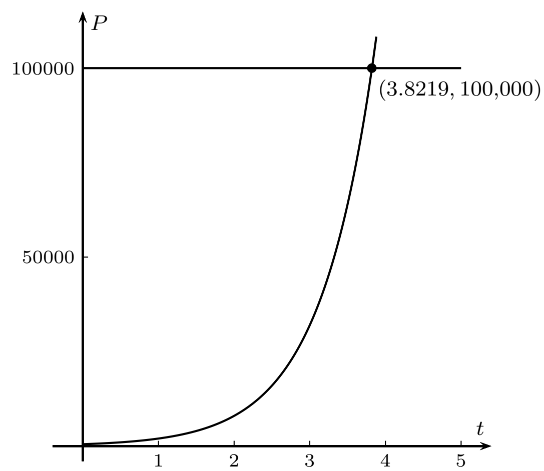
(c) *How many bacteria are there after 40 minutes?*

The input t to our function is given in hours, so we must convert 40 minutes into hours. So $(40\,\text{min})\left(\frac{1\,\text{hr}}{60\,\text{min}}\right) = \frac{2}{3}\,\text{hr}$.

$$\begin{aligned} P(t) &= 500\mathrm{e}^{1.3863t} \\ P\left(\frac{2}{3}\right) &= 500\mathrm{e}^{1.3863\cdot(2/3)} \\ P\left(\frac{2}{3}\right) &\approx 1259.9258 \end{aligned}$$

Thus, after 40 minutes (2/3 of an hour), there are about 1260 bacteria.

(d) *Graph the population function and estimate the time for the population to reach* 100,000.



Using the `calc:intersect` on the TI-84, we get the point $(3.8219, 100000)$, thus the population will reach 100,000 in about 3.82 yrs.