When a camera flash goes off, the batteries immediately begin to recharge the flash's capacitor, which stores electric charge given by

$$Q(t) = Q_0\left(1 - e^{-t/a}\right)$$

(The maximum charge capacity is Q_0 and t is measured in seconds.)[1]

(a) *Find the inverse of this function and explain its meaning.*

$Q(t) = Q_0\left(1 - e^{-t/a}\right)$ gives charge as a function of time. Thus the inverse will give time as a function of charge. That is, when we input a specific amount of charge, the output will tell us the time when that charge was reached.

$$Q = Q_0\left(1 - e^{-t/a}\right)$$

and we solve for t

$$\begin{aligned}\frac{Q}{Q_0} &= 1 - e^{-t/a}\\ \frac{Q}{Q_0} - 1 &= -e^{-t/a}\\ -\frac{Q}{Q_0} + 1 &= e^{-t/a}\end{aligned}$$

taking the natural log of both sides

$$\begin{aligned}\ln\left(1 - \frac{Q}{Q_0}\right) &= \ln\left(e^{-t/a}\right)\\ \ln\left(1 - \frac{Q}{Q_0}\right) &= -t/a\\ -a \cdot \ln\left(1 - \frac{Q}{Q_0}\right) &= t\end{aligned}$$

So, in function form,

$$t = T(Q) = -a\ln\left(1 - \frac{Q}{Q_0}\right)$$

(b) *How long does it take to recharge the capacitor to 90% of capacity if $a = 2$?*

We substitute $a = 2$ and $Q = 0.90Q_0$ into the inverse function.

$$\begin{aligned}T(Q) &= -2\ln\left(1 - \frac{Q}{Q_0}\right)\\ T(0.90Q_0) &= -2\ln\left(1 - \frac{0.90Q_0}{Q_0}\right)\\ T(0.90Q_0) &= -2\ln(1 - 0.90)\\ T(0.90Q_0) &= -2\ln(0.10)\\ T(0.90Q_0) &\approx 4.6052\end{aligned}$$

Thus the charge will be at 90% after about 4.6 sec.

[1] Stewart, *Calculus, Early Transcendentals*, p. 68, #62.