

Calculus I, Section 2.1, #8  
The Tangent and Velocity Problems

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The displacement (in centimeters) of a particle moving back and forth along a straight line is given by the equation of motion  $s = 2 \sin(\pi t) + 3 \cos(\pi t)$ , where  $t$  is measured in seconds.<sup>1</sup>

(a) Find the average velocity during each time period.

We will use the TI-84 for calculation. The units on the average velocity will be  $\frac{\text{cm}}{\text{sec}}$ .

(i) [1,2]

$$\begin{aligned}\text{ave. vel.} &= \frac{s(2) - s(1)}{2 - 1} \\ &= 6 \frac{\text{cm}}{\text{sec}}\end{aligned}$$

(ii) [1,1.1]

$$\begin{aligned}\text{ave. vel.} &= \frac{s(1.1) - s(1)}{1.1 - 1} \\ &\approx -4.7120 \frac{\text{cm}}{\text{sec}}\end{aligned}$$

(iii) [1,1.01]

$$\begin{aligned}\text{ave. vel.} &= \frac{s(1.01) - s(1)}{1.01 - 1} \\ &\approx -6.1341 \frac{\text{cm}}{\text{sec}}\end{aligned}$$

(iv) [1,1.001]

$$\begin{aligned}\text{ave. vel.} &= \frac{s(1.001) - s(1)}{1.001 - 1} \\ &\approx -6.2684 \frac{\text{cm}}{\text{sec}}\end{aligned}$$

(b) Estimate the instantaneous velocity of the particle when  $t = 1$ .

Let's do one more calculation of average velocity over the interval [1,1.0001].

$$\begin{aligned}\text{ave. vel.} &= \frac{s(1.0001) - s(1)}{1.0001 - 1} \\ &\approx -6.2817 \frac{\text{cm}}{\text{sec}}\end{aligned}$$

This, together with the computations from part (a), lead us to estimate the instantaneous velocity at  $t = 1$  to be  $\approx -2\pi \frac{\text{cm}}{\text{sec}}$ .

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<sup>1</sup>Stewart, *Calculus, Early Transcendentals*, p. 82, #8.