(a) *Evaluate* $h(x) = (\tan(x) - x)/x^3$ *for* $x = 1, 0.5, 0.1, 0.05, 0.01,$ *and* 0.005.[1]

x	$(\tan(x) - x)/x^3$
1	0.5574
0.5	0.3704
0.1	0.3347
0.05	0.3337
0.01	0.3333
0.005	0.3333

(b) *Guess the value of* $\lim_{x\to 0} \frac{\tan(x)-x}{x^3}$.

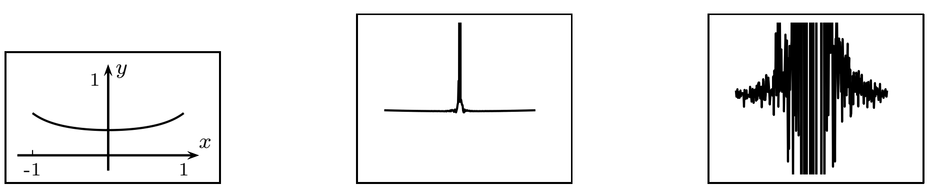
From the table, it seems that $\lim_{x\to 0} \frac{\tan(x)-x}{x^3} = \frac{1}{3}$.

(c) *Evaluate* $h(x)$ *for successively smaller values of* x *until you finally reach a value of* 0 *for* $h(x)$. *Are you still confident that your guess in part (b) is correct? Explain why you eventually reached* 0 *values. (We will learn a method–l'Hospital's Rule–for evaluating this limit in a future lesson.)*

x	$(\tan(x) - x)/x^3$
0.001	0.3333335
0.0001	0.33333
0.00001	0.333
0.000001	0
0.0000001	0

It seems that the limit is now zero! We suspect that the calculator is evaluating the subtraction in the numerator and getting a number close to zero, forcing the calculator's approximation to be zero. (In fact, with very small numbers, this subtraction is a problem with *all* computing devices.)

(d) *Graph the function* h *in the viewing rectangle* $[-1,1]$ *by* $[0,1]$. *Then zoom in toward the point where the graph crosses the* y*-axis to estimate the limit of* $h(x)$ *as* x *approaches* 0. *Continue to zoom in until you observe distortions in the graph of* h. *Compare with the results of part (c).*



Clearly, the computer (or calculator) is having a great deal of difficulty with the graph of this function near zero.

Note: We will see later that this limit is indeed $\frac{1}{3}$. This problem should leave you wary of using technology without being fully aware of how that technology is getting the results it displays.

[1] Stewart, *Calculus, Early Transcendentals*, p. 94, #50.