Evaluate the limit, if it exists.[1]

$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)$$

If we try direct substitution, we get fractions that are undefined. Let's do the algebra to rewrite the difference as a single fraction.

$$\begin{aligned}\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right) &= \lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t\,(t+1)}\right)\\ &= \lim_{t\to 0}\left(\frac{1}{t}\cdot\frac{t+1}{t+1}-\frac{1}{t\,(t+1)}\right)\\ &= \lim_{t\to 0}\left(\frac{t+1}{t\,(t+1)}-\frac{1}{t\,(t+1)}\right)\\ &= \lim_{t\to 0}\left(\frac{t+1-1}{t\,(t+1)}\right)\\ &= \lim_{t\to 0}\left(\frac{t}{t\,(t+1)}\right)\\ &= \lim_{t\to 0}\frac{1}{(t+1)}\end{aligned}$$

and now we do direct substitution

$$\begin{aligned}&= \frac{1}{(0+1)}\\ &= 1\end{aligned}$$

Thus,

$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)=1$$

[1]Stewart, *Calculus, Early Transcendentals*, p. 103, #26.