In the theory of relativity, the Lorentz contraction formula

$$L = L_0\sqrt{1 - v^2/c^2}$$

expresses the length L of an object as a function of its velocity v with respect to an observer, where L_0 is the length of the object at rest and c is the speed of light. Find $\lim_{v\to c^-} L$ *and interpret the result. Why is a left hand limit necessary?*[1]

A left hand limit is necessary because the velocity cannot be greater than the speed of light.

$$\begin{aligned}\lim_{v\to c^-} L_0\sqrt{1-\frac{v^2}{c^2}} &= \lim_{v\to c^-} L_0\sqrt{\frac{c^2}{c^2}-\frac{v^2}{c^2}} \\ &= \lim_{v\to c^-} L_0\sqrt{\frac{c^2-v^2}{c^2}}\end{aligned}$$

and since we know $c > 0$

$$\begin{aligned}&= \lim_{v\to c^-} L_0\frac{\sqrt{c^2-v^2}}{c} \\ &= \lim_{v\to c^-} \frac{L_0}{c}\sqrt{c^2-v^2}\end{aligned}$$

Now, as $v \to c^-$, $v^2 \to c^2$, and $c^2 - v^2 \to 0$. So

$$= 0$$

Thus, $\lim_{v\to c^-} L_0\sqrt{1-\frac{v^2}{c^2}} = 0$.

As the object's velocity increases towards c, it's length decreases towards 0.

[1]Stewart, *Calculus, Early Transcendentals*, p. 104, #56.