A crystal growth furnace is used in research to determine how to best manufacture crystals used in electronic components for the space shuttle. For proper growth of the crystal, the temperature must be controlled accurately by adjusting the input power. Suppose the relationship is given by

$$T(w) = 0.1w^2 + 2.155w + 20$$

where T is the temperature in degrees Celsius and w is the power input in watts.[1]

(a) *How much power is needed to maintain the temperature at* 200° C*?*

We need to solve $T(w) = 0.1w^2 + 2.155w + 20 = 200$.

$$\begin{aligned} 0.1w^2 + 2.155w + 20 &= 200 \\ 0.1w^2 + 2.155w - 180 &= 0 \end{aligned}$$

Using the quadratic formula,

$$w = \frac{-2.155 \pm \sqrt{(2.155)^2 - 4 \cdot 0.1 \cdot -180}}{2 \cdot 0.1}$$

so

$$w \approx 32.9983 \quad \text{or} \quad w \approx -54.5483$$

Since power is a positive quantity, we take $w = 33$.

(b) *If the temperature is allowed to vary from* 200° C *by up to* $\pm 1^\circ$ C*, what range of wattage is allowed for the input power?*

We need to find the wattage for a temperature of 199° C and the wattage for a temperature of 201° C. Using the same analysis we did in part (a), we get $w \approx 32.8839$ when $t = 199^\circ$ C and $w \approx 33.1124$ when $t = 201^\circ$ C

From part (a), we know that $w = 33$ so the two possible values of δ are $\delta_1 = |32.8839 - 33| = 0.1161$ and $\delta_2 = |33.1124 - 33| = 0.1124$. We choose the smallest of these two, so $\delta = 0.1124$

(c) *In terms of the ϵ, δ definition of* $\lim_{x \to a} f(x) = L$*, what is x? What is $f(x)$? What is a? What is L? What value of ϵ is given? What is the corresponding value of δ?*

Here x is the input power w and $f(x)$ is the temperature T. a is the required power to make L the desired temperature of 200° C. ϵ is the tolerance allowed for the temperature, $\pm 1^\circ$ C. Finally, δ is the tolerance for the power input; from part (b), $\delta = 0.1124$.

[1] Stewart, *Calculus, Early Transcendentals*, p. 113, #12.