*Prove the statement using the $\epsilon$, $\delta$ definition of a limit.*[1]

$$\lim_{x \to -1.5} \frac{9-4x^2}{3+2x} = 6$$

Given $\epsilon > 0$, we need $\delta > 0$ such that if $0 < |x - (-1.5)| < \delta$, then $\left| \frac{9-4x^2}{3+2x} - 6 \right| < \epsilon$.

To find the appropriate $\delta$, we start with the relationship for epsilon, and do whatever algebra is needed to find the relationship for $\delta$.[2]

$$\begin{aligned}
\left| \frac{9-4x^2}{3+2x} - 6 \right| &< \epsilon \\
\left| \frac{(3-2x)(3+2x)}{3+2x} - 6 \right| &< \epsilon \\
|3-2x-6| &< \epsilon \\
|-3-2x| &< \epsilon \\
|-2| \left| \frac{3}{2} + x \right| &< \epsilon \\
2 \cdot \left| x + \frac{3}{2} \right| &< \epsilon \\
\left| x + \frac{3}{2} \right| &< \frac{\epsilon}{2} \qquad \text{(Note that } \left| x + \tfrac{3}{2} \right| = |x-(-1.5)|.\text{)}
\end{aligned}$$

Since all of these steps are reversible, we now know to choose $\delta = \frac{\epsilon}{2}$. We construct the actual proof[3], beginning with $0 < |x + 1.5| < \delta$ and leading to $\left| \frac{9-4x^2}{3+2x} - 6 \right| < \epsilon$.

Choose $\delta = \frac{\epsilon}{2}$. We have

$$\begin{aligned}
&0 < |x+1.5| < \delta \\
&\implies 0 < |x+1.5| < \frac{\epsilon}{2} \\
&\implies |-2| \left| \frac{3}{2} + x \right| < \epsilon \\
&\implies |-3-2x| < \epsilon \\
&\implies \left| \frac{(3-2x)(3+2x)}{3+2x} - 6 \right| < \epsilon \qquad \text{(Since } 0 < |x-(-1.5)|,\ x \neq -\tfrac{3}{2}, \text{ and } 3+2x \neq 0.\text{)} \\
&\implies \left| \frac{9-4x^2}{3+2x} - 6 \right| < \epsilon
\end{aligned}$$

Thus, given $\epsilon > 0$, we have $\delta = \frac{\epsilon}{2} > 0$ such that if $0 < |x-(-1.5)| < \delta$, then $\left| \frac{9-4x^2}{3+2x} - 6 \right| < \epsilon$.

[1] Stewart, *Calculus, Early Transcendentals*, p. 114, #22.
[2] Many textbooks refer to this as "Guessing a value for $\delta$."
[3] Many textbooks refer to this as "Showing that this $\delta$ works."