Prove the statement using the ϵ, δ definition of a limit.[1]

$$\lim_{x\to -6^+} \sqrt[8]{6+x} = 0$$

Given $\epsilon > 0$, we need $\delta > 0$ such that if $0 < |x-(-6)| < \delta$, then $\left|\sqrt[8]{6+x} - 0\right| < \epsilon$.

To find the appropriate δ, we start with the relationship for epsilon, and do whatever algebra is needed to find the relationship for δ.[2]

$$\begin{aligned}
\left|\sqrt[8]{6+x} - 0\right| &< \epsilon \\
\left|\sqrt[8]{6+x}\right| &< \epsilon \\
\sqrt[8]{6+x} &< \epsilon \qquad \text{(Since } x \to -6^+ \text{ we know } x > -6 \text{, and } \sqrt[8]{6+x} > 0.) \\
6+x &< \epsilon^8 \\
x-(-6) &< \epsilon^8
\end{aligned}$$

Since all of these steps are reversible, we now know to choose $\delta = \epsilon^8$. We construct the actual proof[3], beginning with $0 < |x-(-6)| < \delta$ and leading to $\left|\sqrt[8]{6+x} - 0\right| < \epsilon$.

Choose $\delta = \epsilon^8$. We have

$$\begin{aligned}
&0 < |x-(-6)| < \delta \\
&\implies 0 < |x-(-6)| < \epsilon^8 \\
&\implies \sqrt[8]{|x-(-6)|} < \epsilon \\
&\implies \left|\sqrt[8]{x-(-6)}\right| < \epsilon \qquad \text{(Since we know } \sqrt[8]{x-(-6)} > 0.) \\
&\implies \left|\sqrt[8]{x+6}\right| < \epsilon \\
&\implies \left|\sqrt[8]{x+6} - 0\right| < \epsilon
\end{aligned}$$

Thus, given $\epsilon > 0$, we have $\delta = \epsilon^8 > 0$ such that if $0 < |x-(-6)| < \delta$, then $\left|\sqrt[8]{6+x} - 0\right| < \epsilon$.

[1] Stewart, *Calculus, Early Transcendentals*, p. 114, #28.
[2] Many textbooks refer to this as "Guessing a value for δ."
[3] Many textbooks refer to this as "Showing that this δ works."