Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.[1]

$$f(x) = 3x^4 - 5x + \sqrt[3]{x^2 + 4}, \quad a = 2$$

We have the following definitions:

> **Definition** A function f is **continuous at a number a** if
>
> $$\lim_{x \to a} f(x) = f(a)$$

> **Definition** A function f is **continuous on an interval** if it is continuous at every number in the interval. (If f is defined only on one side of an endpoint of the interval, we understand *continuous* at the endpoint to men *continuous from the right* or *continuous from the left.*)

We'll focus on the first definition because this problem asks about continuity at the particular number $a = 2$.

We apply the properties of limits

$$\begin{aligned}
\lim_{x \to 2} f(x) &= \lim_{x \to 2} 3x^4 - 5x + \sqrt[3]{x^2 + 4} \\
&= \lim_{x \to 2} 3x^4 - \lim_{x \to 2} 5x + \lim_{x \to 2} \sqrt[3]{x^2 + 4} \\
&= 3 \lim_{x \to 2} x^4 - 5 \lim_{x \to 2} x + \lim_{x \to 2} \sqrt[3]{x^2 + 4} \\
&= 3 \left(\lim_{x \to 2} x\right)^4 - 5 \lim_{x \to 2} x + \sqrt[3]{\lim_{x \to 2} (x^2 + 4)} \\
&= 3 \left(\lim_{x \to 2} x\right)^4 - 5 \lim_{x \to 2} x + \sqrt[3]{\lim_{x \to 2} x^2 + \lim_{x \to 2} 4} \\
&= 3 \left(\lim_{x \to 2} x\right)^4 - 5 \lim_{x \to 2} x + \sqrt[3]{\left(\lim_{x \to 2} x\right)^2 + \lim_{x \to 2} 4} \\
&= 3 \cdot 2^4 - 5 \cdot 2 + \sqrt[3]{2^2 + 4} \\
&= 40
\end{aligned}$$

But

$$\begin{aligned}
f(2) &= 3 \cdot 2^4 - 5 \cdot 2 + \sqrt[3]{2^2 + 4} \\
&= 40
\end{aligned}$$

Thus,

$$\lim_{x \to 2} 3x^4 - 5x + \sqrt[3]{x^2 + 4} = f(2)$$

so the function f is continuous at $x = 2$.

[1]Stewart, *Calculus, Early Transcendentals*, p. 124, #14.