

Calculus I, Section 2.5, #60
Continuity

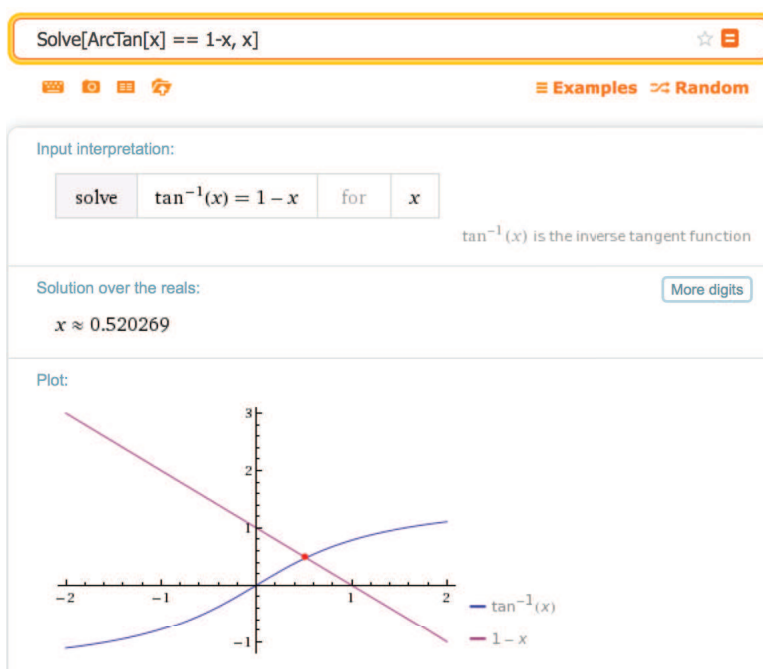
(a) Prove that the equation has at least one real root. (b) Use your graphing device to find the root correct to three decimal places.¹

$$\arctan(x) = 1 - x$$

To prove that the equation has at least one real root, we will rewrite the equation as a function, then find a value of x that makes the function negative, and one that makes the function positive.

Let $f(x) = \arctan(x) - 1 + x$. Note that $f(0) = \arctan(0) - 1 + 0 = -1$ and $f(1) = \arctan(1) - 1 + 1 = \frac{\pi}{4}$. The function f is continuous because it is the sum or difference of a continuous inverse trig function and a polynomial. Thus, from the Intermediate Value Theorem, there must be at least one real number in the interval $(0,1)$ for which $\arctan(x) - 1 + x = 0$.

Using WolframAlpha,



so the solution to $\arctan(x) = 1 - x$ is $x \approx 0.520$

¹Stewart, *Calculus, Early Transcendentals*, p. 126, #60.