Find the limit or show that it does not exist.[1]

$$\lim_{x\to-\infty} \frac{\sqrt{1+4x^6}}{2-x^3}$$

$$\lim_{x\to-\infty} \frac{\sqrt{1+4x^6}}{2-x^3} = \lim_{x\to-\infty} \frac{\frac{\sqrt{1+4x^6}}{x^3}}{\frac{2-x^3}{x^3}}$$

Since $x \to -\infty$, we know x is negative, so $x^3 = -\sqrt{x^6}$. Substituting

$$= \lim_{x\to-\infty} \frac{\frac{\sqrt{1+4x^6}}{-\sqrt{x^6}}}{\frac{2}{x^3} - \frac{x^3}{x^3}}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{\frac{1+4x^6}{x^6}}}{\frac{2}{x^3} - 1}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{\frac{1}{x^6} + \frac{4x^6}{x^6}}}{\frac{2}{x^3} - 1}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{\frac{1}{x^6} + 4}}{\frac{2}{x^3} - 1}$$

and since $\lim_{x\to-\infty} \frac{k}{x^r} = 0$, for k a constant,

$$= \frac{-\sqrt{0+4}}{0-1}$$

$$= 2$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 138, #24.