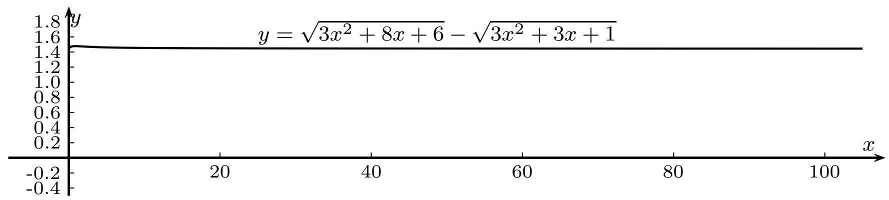
(a) *Use a graph of*

$$f(x) = \sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1}$$

to estimate the value of $\lim_{x\to\infty} f(x)$ *to one decimal place.*[1]



From the graph, it seems $\lim_{x\to\infty} \sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1} \approx 1.4$.

(b) Use a table of values of $f(x)$ to estimate the limit to four decimal places.

x	$\sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1}$
1	1.47735432
10	1.45347731
100	1.44455675
1000	1.44349573
10,000	1.44338770
100,000	1.44337688

From the table, it seems $\lim_{x\to\infty} \sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1} \approx 1.4433$.

(c) Find the exact value of the limit.

We want

$$\lim_{x\to\infty} \sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1}$$

Direct substitution is not possible because ∞ is *not* a real number. Because there is a difference of roots, let's try multiplying numerator and denominator by the conjugate.

$$\begin{aligned}
&\lim_{x\to\infty} \sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1} \\
&\quad = \lim_{x\to\infty} \frac{\left(\sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1}\right)\left(\sqrt{3x^2 + 8x + 6} + \sqrt{3x^2 + 3x + 1}\right)}{\sqrt{3x^2 + 8x + 6} + \sqrt{3x^2 + 3x + 1}} \\
&\quad = \lim_{x\to\infty} \frac{\left(3x^2 + 8x + 6\right) - \left(3x^2 + 3x + 1\right)}{\sqrt{3x^2 + 8x + 6} + \sqrt{3x^2 + 3x + 1}} \\
&\quad = \lim_{x\to\infty} \frac{5x + 5}{\sqrt{3x^2 + 8x + 6} + \sqrt{3x^2 + 3x + 1}} \\
&\quad = \lim_{x\to\infty} \frac{(5x + 5)\left(\frac{1}{x}\right)}{\left(\sqrt{3x^2 + 8x + 6} + \sqrt{3x^2 + 3x + 1}\right)\left(\frac{1}{x}\right)}
\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 138, #46.

$$= \lim_{x\to\infty} \frac{5 + \frac{5}{x}}{\sqrt{\frac{3x^2}{x^2} + \frac{8x}{x^2} + \frac{6}{x^2}} + \sqrt{\frac{3x^2}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{5 + \frac{5}{x}}{\sqrt{3 + \frac{8}{x} + \frac{6}{x^2}} + \sqrt{3 + \frac{3}{x} + \frac{1}{x^2}}}$$

$$= \frac{5}{\sqrt{3} + \sqrt{3}}$$

$$= \frac{5}{2\sqrt{3}}$$