If a rock is thrown upward on the planet Mars with a velocity of $10\,\mathrm{m/s}$*, its height (in meters) after* t *seconds is given by* $H = 10t - 1.86t^2$*.*[1]

(a) *Find the velocity of the rock after one second.*

$$\begin{aligned}
v(1) &= \lim_{h\to 0} \frac{H(1+h) - H(1)}{h} \\
&= \lim_{h\to 0} \frac{\left(10\,(1+h) - 1.86\,(1+h)^2\right) - \left(10\cdot 1 - 1.86\cdot 1^2\right)}{h} \\
&= \lim_{h\to 0} \frac{10 + 10h - 1.86\left(1 + 2h + h^2\right) - 10 + 1.86}{h} \\
&= \lim_{h\to 0} \frac{10 + 10h - 1.86 - 3.72h - 1.86h^2 - 10 + 1.86}{h} \\
&= \lim_{h\to 0} \frac{6.28h - 1.86h^2}{h} \\
&= \lim_{h\to 0} \frac{h\,(6.28 - 1.86h)}{h} \\
&= \lim_{h\to 0} (6.28 - 1.86h) \\
&= 6.28
\end{aligned}$$

Thus the velocity of the rock after one second is $6.28\,\mathrm{m/s}$.

(b) *Find the velocity of the rock when* $t = a$*.*

$$\begin{aligned}
v(a) &= \lim_{h\to 0} \frac{H(a+h) - H(a)}{h} \\
&= \lim_{h\to 0} \frac{\left(10\,(a+h) - 1.86\,(a+h)^2\right) - \left(10a - 1.86a^2\right)}{h} \\
&= \lim_{h\to 0} \frac{10a + 10h - 1.86\left(a^2 + 2ah + h^2\right) - 10a + 1.86a^2}{h} \\
&= \lim_{h\to 0} \frac{10a + 10h - 1.86a^2 - 3.72ah - 1.86h^2 - 10a + 1.86a^2}{h} \\
&= \lim_{h\to 0} \frac{10h - 3.72ah - 1.86h^2}{h} \\
&= \lim_{h\to 0} \frac{h\,(10 - 3.72a - 1.86h)}{h} \\
&= \lim_{h\to 0} (10 - 3.72a - 1.86h) \\
&= 10 - 3.72a
\end{aligned}$$

Thus, at $t = a$, the velocity of the rock is $10 - 3.72\,\mathrm{m/s}$.

(Note that if $a = 1$, this result gives the same value as the computation in part (a).)

[1] Stewart, *Calculus, Early Transcendentals*, p. 149, #14.

(c) *When will the rock hit the surface?*

We solve

$$\begin{aligned} 0 &= 10t - 1.86t^2 \\ 0 &= t(10 - 1.86t) \end{aligned}$$

and by the zero product property

$$0 = t \quad \text{or} \quad 0 = 10 - 1.86t$$

so

$$0 = t \quad \text{or} \quad 5.38 \approx t$$

Thus the rock hits the surface after about 5.38 s.

(d) *With what velocity will the rock hit the surface?*

Using our result from part (b),

$$\begin{aligned} v(a) &= 10 - 3.72a \\ v(5.38) &= 10 - 3.72 \cdot 5.38 \\ v(5.38) &\approx -10.01 \end{aligned}$$

Thus the rock hits the surface with a velocity of about -10.01 m/s. Here, the negative value indicates that the rock is traveling downward.