

Calculus I, Section 2.7, #48
 Derivatives and Rates of Change

The number N of locations of a popular coffeehouse chain is given in the table. (The numbers of locations as of October 1 are given.)¹

Year	2004	2006	2008	2010	2012
N	8569	12,440	16,680	16,858	18,066

(a) Find the average rate of growth

(i) from 2006 to 2008

(ii) from 2008 to 2010

In each case, include the units. What can you conclude?

$$\begin{aligned} \text{AROC}_{2006 \rightarrow 2008} &= \frac{16680 - 12440}{2008 - 2006} \\ &= 2120 \frac{\text{locations}}{\text{year}} \end{aligned}$$

$$\begin{aligned} \text{AROC}_{2008 \rightarrow 2010} &= \frac{16858 - 16680}{2010 - 2008} \\ &= 89 \frac{\text{locations}}{\text{year}} \end{aligned}$$

The number of locations continues to increase (both AROCs are positive), but the growth was much slower from 2008 to 2010.

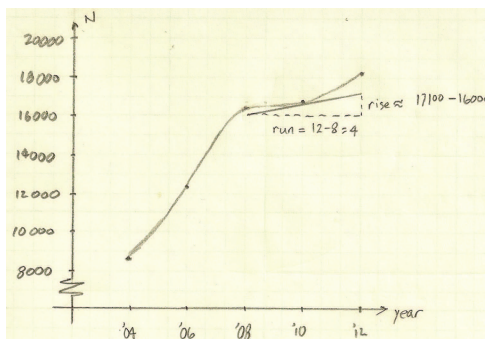
(b) Estimate the instantaneous rate of growth in 2010 by taking the average of two average rates of change. What are its units?

$$\begin{aligned} \text{AROC}_{2008 \rightarrow 2010} &= \frac{16858 - 16680}{2010 - 2008} \\ &= 89 \frac{\text{locations}}{\text{year}} \end{aligned}$$

$$\begin{aligned} \text{AROC}_{2010 \rightarrow 2012} &= \frac{18066 - 16858}{2012 - 2010} \\ &= 604 \frac{\text{locations}}{\text{year}} \end{aligned}$$

$$\text{IROC}_{2010} \approx \frac{89 \frac{\text{locations}}{\text{year}} + 604 \frac{\text{locations}}{\text{year}}}{2} = 346.5 \frac{\text{locations}}{\text{year}}$$

(c) Estimate the instantaneous rate of growth in 2010 by measuring the slope of a tangent.



From the triangle on the diagram, $N'(10) \approx \frac{17100 - 16000}{2012 - 2008} = 275 \frac{\text{locations}}{\text{year}}$.

¹Stewart, *Calculus, Early Transcendentals*, p. 150, #48.