

Calculus I, Section 3.1, #22
Derivatives of Polynomials and Exponential Functions

Differentiate the function.¹

$$y = \frac{\sqrt{x} + x}{x^2}$$

We will use algebra to rewrite the given function in the form of the power rule for differentiation.

$$\begin{aligned} y &= \frac{\sqrt{x} + x}{x^2} \\ &= \frac{\sqrt{x}}{x^2} + \frac{x}{x^2} \\ &= \frac{x^{1/2}}{x^2} + \frac{x}{x^2} \\ &= x^{1/2-2} + x^{1-2} \\ &= x^{-3/2} + x^{-1} \end{aligned}$$

Now that the function is expressed with powers, we can use the power rule.

$$\begin{aligned} y' &= -\frac{3}{2}x^{-3/2-1} + -1 \cdot x^{-1-1} \\ &= -\frac{3}{2}x^{-5/2} - x^{-2} \end{aligned}$$

or

$$\begin{aligned} &= -\frac{3}{2} \cdot \frac{1}{x^{5/2}} - \frac{1}{x^2} \\ &= -\frac{3}{2x^{5/2}} - \frac{1}{x^2} \end{aligned}$$

or

$$= -\frac{3}{2\sqrt{x^5}} - \frac{1}{x^2}$$

or

$$\begin{aligned} &= -\frac{3\sqrt{x^5}}{2x^5} - \frac{1}{x^2} \\ &= -\frac{3\sqrt{x^5}}{2x^5} - \frac{1}{x^2} \cdot \frac{2x^3}{2x^3} \\ &= -\frac{3\sqrt{x^5}}{2x^5} - \frac{2x^3}{2x^5} \\ &= -\frac{3\sqrt{x^5} + 2x^3}{2x^5} \end{aligned}$$

Which of these is correct? They all are! The final form of the result is determined by (1) how the result will be used, and (2) what your teacher expects.

¹Stewart, *Calculus, Early Transcendentals*, p. 180, #22.