

Calculus I, Section 3.1, #54
Derivatives of Polynomials and Exponential Functions

Car tires need to be inflated properly because overinflation or underinflation can cause premature tread wear. The data in the table show tire life L (in thousands of miles) for a certain type of tire at various pressures P (in lb/in^2).¹

P	26	28	31	35	38	42	45
L	50	66	78	81	74	70	59

- (a) Use a calculator to model tire life with a quadratic function of the pressure.

Here, the input to the function is the pressure and the output is the tire life. Using the TI-84, we get

$$L(P) = -0.2754P^2 + 19.7485P - 273.5523$$

(Step-by-step instructions for the data analysis can be found in the document “6 Important Graphing Calculator Skills”.)

- (b) Use the model to estimate dL/dP when $P = 30$ and when $P = 40$. What is the meaning of the derivative? What are the units? What is the significance of the signs of the derivatives?

$$\frac{dL}{dP} = -0.5508P + 19.7485$$

so

$$\begin{aligned}\left.\frac{dL}{dP}\right|_{P=30} &= -0.5508(30) + 19.7485 \\ &\approx 3.22\end{aligned}$$

$$\begin{aligned}\left.\frac{dL}{dP}\right|_{P=40} &= -0.5508(40) + 19.7485 \\ &\approx -2.28\end{aligned}$$

Here, the derivative gives the rate of change of tire life as a function of the pressure. The units are $\frac{\text{thousands of miles}}{\text{lb}/\text{in}^2}$. (As with all derivatives, the units are $\frac{\text{units of output from original function}}{\text{units of input to original function}}$.) At $P = 30$, the derivative is positive, so tire life is increasing, while at $P = 40$ the derivative is negative, so tire life is decreasing.

¹Stewart, *Calculus, Early Transcendentals*, p. 181, #54.