Find a parabola with equation $y = ax^2 + bx + c$ that has slope 4 at $x = 1$, slope -8 at $x = -1$, and passes through the point (2,15).[1]

"...slope 4 at $x = 1$, slope -8 at $x = -1$ ..." tells us about the value of the derivative.

$$y' = 2ax + b$$

Substituting slope 4 at $x = 1$,

$$4 = 2a \cdot 1 + b$$
$$4 = 2a + b$$

Substituting slope -8 at $x = -1$,

$$-8 = 2a \cdot -1 + b$$
$$-8 = -2a + b$$

Now we solve the system of equations $\begin{cases} 2a + b = 4 \\ -2a + b = -8 \end{cases}$

If we add the equations, we get $2b = -4$, so $b = -2$.

If we subtract the equations, we get $4a = 12$, so $a = 3$.

"...and passes through the point (2,15)." means the point (2,15) is on the graph of the parabola, and is a solution to the equation. Substituting,

$$y = ax^2 + bx + c$$
$$15 = a\,(2)^2 + b\,(2) + c$$
$$15 = 4a + 2b + c$$

Substituting our values for a and b gives us

$$15 = 4 \cdot 3 + 2 \cdot -2 + c$$
$$7 = c$$

Thus the parabola with equation $y = ax^2 + bx + c$ that has slope 4 at $x = 1$, slope -8 at $x = -1$, and passes through the point (2,15) is

$$y = 3x^2 - 2x + 7$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 181, #70.