

Calculus I, Section 3.2, #36
The Product and Quotient Rules

- (a) The curve $y = x/(1+x^2)$ is called a *serpentine*. Find an equation of the tangent line to this curve at the point $(3,0.3)$.¹

To write the equation of the tangent line, we need the slope and a point on the line. The slope will come from the derivative, and we are given the point $(3,0.3)$ on the tangent line.

If $y = x/(1+x^2)$, then

$$\begin{aligned}y' &= \frac{(1+x^2) \cdot 1 - x \cdot 2x}{(1+x^2)^2} \\ &= \frac{1+x^2-2x^2}{(1+x^2)^2} \\ &= \frac{1-x^2}{(1+x^2)^2}\end{aligned}$$

$$\text{When } x = 3, y' = \frac{1-3^2}{(1+3^2)^2} = \frac{-8}{100} = -\frac{2}{25}.$$

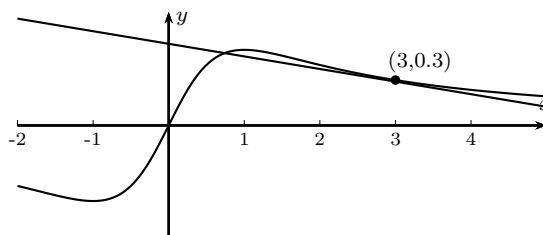
Using the point-slope form for the equation of a line, we have

$$y - 0.3 = -\frac{2}{25}(x - 3)$$

and in slope-intercept form,

$$y = -\frac{2}{25}x + \frac{27}{50}$$

- (b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.



¹Stewart, *Calculus, Early Transcendentals*, p. 188, #36.