

Calculus I, Section 3.4, #26
The Chain Rule

Find the derivative of the function.¹

$$s(t) = \sqrt{\frac{1 + \sin(t)}{1 + \cos(t)}}$$

$$s(t) = \sqrt{\frac{1 + \sin(t)}{1 + \cos(t)}} = \left(\frac{1 + \sin(t)}{1 + \cos(t)}\right)^{1/2}$$

← outside function
inside function

$$\begin{aligned} \frac{ds}{dt} &= \frac{1}{2} \left(\frac{1 + \sin(t)}{1 + \cos(t)}\right)^{1/2-1} \cdot \frac{(1 + \cos(t)) \cdot \cos(t) - (1 + \sin(t)) \cdot -\cos(t)}{(1 + \cos(t))^2} \\ &= \frac{1}{2} \left(\frac{1 + \sin(t)}{1 + \cos(t)}\right)^{-1/2} \frac{\cos(t) + \cos^2(t) + \sin(t) + \sin^2(t)}{(1 + \cos(t))^2} \\ &= \frac{1}{2} \frac{(1 + \sin(t))^{-1/2} \cos(t) + \sin(t) + 1}{(1 + \cos(t))^{-1/2} (1 + \cos(t))^2} \\ &= \frac{1}{2} \frac{(1 + \cos(t))^{1/2} \cos(t) + \sin(t) + 1}{(1 + \sin(t))^{1/2} (1 + \cos(t))^2} \\ &= \frac{\cos(t) + \sin(t) + 1}{2\sqrt{1 + \sin(t)} (1 + \cos(t))^{3/2}} \end{aligned}$$

¹Stewart, *Calculus, Early Transcendentals*, p. 204, #26.