**Calculus I, Section 3.4, #84**
**The Chain Rule**

*Under certain circumstances a rumor spreads according to the equation*

$$p(t) = \frac{1}{1 + ae^{-kt}}$$

*where $p(t)$ is the proportion of the population that has heard the rumor at time $t$ and $a$ and $k$ are positive constants.*[1]

(a) *Find* $\lim_{t\to\infty} p(t)$

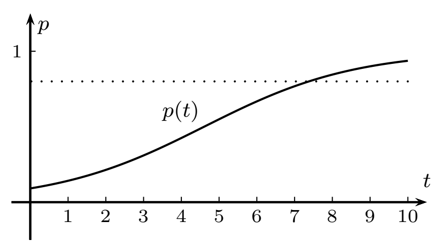
$$\begin{aligned}\lim_{t\to\infty} \frac{1}{1+ae^{-kt}} &= \lim_{t\to\infty} \frac{1}{1+\frac{a}{e^{kt}}} \\ &= \frac{1}{1+0} \qquad \text{Since } a \text{ is constant and } e^{kt} \to 0 \text{ as } t \to \infty \text{ because } k \text{ is positive.} \\ &= 1\end{aligned}$$

(b) *Find the rate of spread of the rumor.*

We want the derivative.

$$\begin{aligned}p'(t) &= \frac{\left(1+ae^{-kt}\right)\cdot 0 - 1\cdot\left(0 + a\cdot e^{-kt}\cdot -k\right)}{\left(1+ae^{-kt}\right)^2} \\ &= \frac{-\left(-ake^{-kt}\right)}{\left(1+ae^{-kt}\right)^2} \\ &= \frac{ake^{-kt}}{\left(1+ae^{-kt}\right)^2}\end{aligned}$$

(c) *Graph $p$ for the case $a = 10$, $k = 0.5$ with $t$ measured in hours. Use the graph to estimate how long it will take 80% of the population to hear the rumor.*




From the graph, it seems that 80% of the population will have heard the rumor in about 7.3 hrs.

[1] Stewart, *Calculus, Early Transcendentals*, p. 206, #84.