Find $\mathrm{d}y/\mathrm{d}x$ *by implicit differentiation.*[1]

$$x\sin(y) + y\sin(x) = 1$$

For the given equation, it is impossible to solve explicitly for y.[2] Since we are asked to find $\mathrm{d}y/\mathrm{d}x$, we treat y as a function of x. This means whenever we take the derivative of y, we apply chain rule and multiply by the derivative of y with respect to x, $\mathrm{d}y/\mathrm{d}x$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[x\sin(y) + y\sin(x)\right] = \frac{\mathrm{d}}{\mathrm{d}x}[1]$$

$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}x}\left[x\sin(y)\right]} + \underbrace{\frac{\mathrm{d}}{\mathrm{d}x}\left[y\sin(x)\right]} = \frac{\mathrm{d}}{\mathrm{d}x}[1]$$

Now we apply product rule

$$\overbrace{x\cdot\cos(y)\cdot\frac{\mathrm{d}y}{\mathrm{d}x} + \sin(y)\cdot 1} + \overbrace{y\cdot\cos(x) + \sin(x)\cdot\frac{\mathrm{d}y}{\mathrm{d}x}} = 0$$

$$x\cos(y)\frac{\mathrm{d}y}{\mathrm{d}x} + \sin(x)\frac{\mathrm{d}y}{\mathrm{d}x} = -\sin(y) - y\cos(x)$$

$$\left(x\cos(y) + \sin(x)\right)\frac{\mathrm{d}y}{\mathrm{d}x} = -\sin(y) - y\cos(x)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-\sin(y) - y\cos(x)}{\left(x\cos(y) + \sin(x)\right)}$$

or

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\sin(y) + y\cos(x)}{x\cos(y) + \sin(x)}$$

[1]Stewart, *Calculus, Early Transcendentals*, p. 215, #18.

[2]Every time we try to get the y out of $\sin(y)$ with inverse sine, the "free" y would then be inside an inverse sine. Then to get that y out of the inverse sine, we apply the sine function, and we're right back where we started.